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ABSTRACT

This guide represents the final experimental version of a pilot project which was conducted in the United States between 1973 and 1976. The ideas and the manner of presentation are based on the works of Georges and Frederique Papy. They are recognized for having introduced colored arrow drawings ("papygrams") and models of our numeration system (the Papy "minicomputer") into the teaching of mathematics at the elementary and secondary level in Belgium. This program follows the "spiral approach." This guide includes the general introduction and the workbooks. Six workbooks are presented which should offer students the opportunity to review ideas they have met before, apply their knowledge to new situations, and learn to read by using mathematics workbooks and story workbooks. The workbooks are written at three levels of difficulty and students are encouraged to work at the easiest level. Questions most frequently asked by teachers about the program are answered in this guide. (Author/SA)

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Comprehensive School Mathematics Program
CEMREL, Inc.

Experimental Version

8-36201

CSMP Mathematics for the Intermediate Grades

Part IV

Teacher's Guide

U.S. DEPARTMENT OF HEALTH,
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General Introduction
Workbooks

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Comprehensive School Mathematics Program

Mathematics for the Intermediate Grades
Part IV

CEMREL, Inc.

PREFACE

CSMP is firmly committed to the proposition that the study of mathematics should be an interesting, challenging, intellectual and aesthetic experience for all children. The teachers who will teach its curriculum should be similarly committed to that proposition. The CSMP curriculum expects to truly engage children in the study of mathematics. It expects to call upon their curiosity, their intuition, their powers of reasoning, their imagination, their perseverance, and their organizational skills. We believe that the study of mathematics not only calls upon these faculties of children, but will help to develop them. But the curriculum cannot do the job by itself -- there must be teachers who share the basic intellectual values built into the curriculum.

There is an almost infinite variety of uses to which mathematics can be and has been put. Certainly, a reasonable mastery of arithmetic skills is part of the mathematical equipment of a competent, modern adult. Our job as educators is to provide children with a set of mathematical experiences far broader than the acquisition of simple arithmetic skills. We believe it is important to expose children to a wide variety of ways in which mathematics can help them to formulate and think about problems. We believe further that the arithmetic skills themselves can be much more successfully taught in the context of a rich and challenging mathematical environment. It is especially important to prepare children with a variety of mathematical activities in view of the fact that no one can possibly know what career and life-style an elementary school child will ultimately want to choose.

It is CSMP's conviction that mathematics is a unified whole and should be learned as such. The organizational basis of the CSMP curriculum is the spiral approach. Thus, the program is sequenced so that students come into contact with each area of content continuously throughout the program. This approach consciously precludes atomizing the content and mastering each bit before continuing to the next. Rather, students encounter repeatedly the ideas, building interlocking experiences of ever increasing sophistication.

CSMP believes that no single method of classroom management suffices to meet the needs of every student. Hence, the program is constructed to allow numerous opportunities for whole class participation, small group participation, and independent individual study.

The CSMP materials are constructed to maximize teacher-student and student-student interaction. Children begin immediately to investigate mathematical concepts in concrete situations appealing and meaningful to them at their level of experience.

Individualization at this level is achieved through graded worksheets and workbooks with problems at varying levels of difficulty all in one booklet. Worksheets and workbooks are integrated into the basic sequence of lessons. Due to the spiral nature of the curriculum, each student can engage in a workbook for a period of two (possibly more) weeks without losing touch with the progress of the class and, at the same time, working his or her way to the limits of his or her capacity.

Unlike most modern programs, the content is not presented as an artificial structure external to the experience of children and into which they are thrust but rather as an extension of experiences children have encountered in their development, both at the real life and fantasy levels. Children are led through sequences of problem solving experiences presented in game-like and other story settings. Nonverbal languages give children immediate access to the mathematical ideas and methods necessary not only for solving these problems but also for continually expanding the students' understanding of the mathematical concepts themselves. These languages include:

1. The language of strings. This language mimics the basic method of organizing and collecting data related to problems at hand. That is, it deals with the fundamentally useful and important mathematical notion of sets.

2. The language of arrows. This graphical language models the fundamental process involved in comparing and analyzing sets and operations on them.

That is, it deals with the useful and important mathematical notions of relations and functions including, but not restricted to, permutations of sets, ordering relations and the numerical processes of adding to, subtracting from, multiplying by, and sharing equally among.

3. The language of the Papy Minicomputer. The Papy Minicomputer, an extremely effective abacus invented by Georges Papy, models the positional structure of our system of numeration and lends itself to very sophisticated algorithmic processes, including the standard algorithms for the basic numerical operations.

Through these languages the children are able to apply the basic mathematical ideas effectively and creatively to explore the content directly without wading through the cumbersome, sterile terminology and notation which, in most other programs, pervade the early stages of the curriculum and stand as barriers between the child and the content.

The content is learned in an atmosphere of constant connection with applications, from simple story situations in the early grades to challenging non-trivial simulations of real world problems in the upper grades. The emphasis is at all times on a two level approach to learning: understanding the content and its applications, and equally important, developing the techniques and processes of learning the content. It is the latter form of knowledge which gives the power to apply the former.

NOTE ON AUTHORSHIP

"Teacher's Guide to CSMP Mathematics for the Intermediate Grades, Part IV" is the product of the concerted efforts of a number of people over several years' time. This edition was produced during the 1977-78 academic year by the following members of the CSMP staff and with substantial contributions from the staff associates.

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The lessons in this guide were developed and tried out with two sixth grade classes at the Brittany Middle School, University City, Missouri. The students, representing a full range of abilities, had been introduced to the CSMP curriculum at the beginning of fourth grade in a one semester program using "CSMP Mathematics for the Intermediate Grades, Entry Module I". The entry program prepared the students to begin the first part of the CSMP Intermediate Grades Program for veteran students during the second semester of the fourth grade.

We are extremely grateful to Mr. Jack Fitzgerald, Mathematics department head at Brittany School, for his cooperation in arranging for CSMP's teaching of the two classes for development purposes. Our greatest motivation for this work was the enthusiasm and steady progress of the students.

GENERAL INTRODUCTION

Comprehensive School Mathematics Program

Mathematics for the Intermediate Grades
Part IV

General Introduction

CEMREL, Inc.

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1. The CSMP Viewpoint

The CSMP curriculum is based on the belief that mathematics belongs to that part of the school curriculum which deals with ideas rather than that part which stresses techniques. This is not to say that there are no techniques involved in the learning of mathematics. There are, just as there are in the learning of English. But these techniques, like spelling and grammar in the learning of English, are not ends in themselves; they are important in so far as they help children to understand and use some area of mathematics.

What are the objects of mathematical study? For us, they include numbers of various kinds; operations on and relationships between numbers; geometrical figures and their properties; relations and functions (e.g., the 2x function, the "less than" relation, etc.) and operations on them.

Among all objects of mathematical study, from the simplest to the most complex, there is one common property: if we separate these objects of study from the real life situations that they model, they are abstract; that is, they are things of the mind. For this reason, mathematics is essentially an intellectual activity. For example, the number 2 represents the phenomenon of twoness, a quality which is present in an endless variety of contexts in life. We can picture 2 by two dots or by a stick two centimeters long, and these representations are certainly useful in suggesting to our minds what may or may not be true about the number 2. But ultimately the number 2 is accessible to us only through our minds. In the same sense, a geometrical line is neither a pencil line nor a stretched string nor a ray of light, but is an abstraction which may be pictured in many ways.

This is surely an adult point of view. But it is one that should be understood and taken into consideration by those who develop mathematical materials for children and those who teach the materials to children.

Since mathematical objects are ideas, one of the ways we explore the properties of and relationships among such objects is to think about them. It is for this reason that growth in the ability to reason plays such an important role in the study of mathematics. We strongly argue, however, that in mathematics the development of the art of reasoning goes hand in hand with the growth of imagination, ingenuity, and intuition. At the elementary level, mathematical objects, their properties, and their interrelationships must be thought about and discussed in various suggestive and interesting situations.

Seen in this perspective, elementary arithmetic then takes the form of "adventures in the world of numbers". Individual numbers such as 0, 1, 2, 100, $\frac{1}{5}$, $^{-}7$, $\frac{^{-}4}{9}$, 3.72, $^{-}1.006$, and so on can assume definite personalities of their own in this world. The teaching of arithmetic shifts its emphasis from an obstacle race for mastery of the standard algorithms to the stimulation of exploring the world of numbers. Skills such as balancing a checkbook have their place in real life but are of absolutely no interest to a fourth or fifth grader. It is unrealistic to view such activities as goals of an elementary school mathematics education. Far more genuinely rewarding are the stretching of the powers of imagination and the challenging of the mind. Facility grows naturally, along with knowledge and understanding. Necessary skills become byproducts of such an educational experience.

Such a curriculum requires a special kind of pedagogy. The pedagogy employed should include teaching strategies which encourage creativity and allow freedom to explore but still foster sound intellectual habits. Attitudes toward student achievement that allow for intellectual and emotional growth and respect the individuality of each student are essential to the successful attainment of the goals of this type of curriculum.

The pedagogy which CSMP develops in this spirit can be described as a "pedagogy of situations".[*] The introductions to the various strands of the CSMP Intermediate Grades Program and those to the chapters within the strands refer to many examples which illustrate aspects of this pedagogy. All these examples are taken from the lessons themselves, and a prime source of

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[*] This phrase is due to the mathematician and psychologist, Caleb Gattegno.

"pedagogical situations" is the lessons of the program. At the risk of oversimplification, a pedagogy of situations may be described as one which is based on the belief that learning occurs in reaction to the experience of confronting a situation (real, simulated, or imagined) that is rich in consequences, worthy of confrontation, and has genuine intellectual content.

Children can react to a situation that intrigues them. They may grapple with problems of symbolization and the relation of a symbol to the thing it represents. Arguments about the consequences of the given information may be close to those which are the stuff of mathematics. Although adults may protest that some of the situations in the lessons are not "real", they are real to children, who can often involve themselves in and feel at home with "pretend" situations more readily than with many "real" situations. On the basis of several years' experience in a wide variety of classes, we can say with confidence that a pedogogy of situations:

- involves children at a personal level (they are interested in the situations presented, and sometimes want to dramatize or devise related situations);
- presents an intellectual challenge that is accessible to a broad range of abilities (even the slowest students understand the challenge and can learn something from their experience in confronting a situation; some may even be able to realize the full learning potential of that challenge);
- provides opportunities for creativity (some children are motivated by well chosen situations to generalize from their experiences or to make conjectures to be tested in similar situations);
- supports experiences that have mathematical content.

The CSMP curriculum fulfills what we consider to be the fundamental role of mathematics in the elementary school by presenting a large number of varied yet interrelated situations that provide the experiences out of which mathematics grows. Two questions immediately arise. How do students learn necessary "number facts" via a pedagogy of situations? And how can we "grade the achievement" of a child as he or she reacts to a situation?

Believe it or not, the "number facts" are among the things which students learn almost inadvertently under a pedagogy of situations. Numbers become personalities; their interrelations are the subject of many stories in which numbers talk to each other and react to each other. In confronting some of these situations, students make various calculations out of necessity. Calculation which serves some higher, more motivating purpose presents nowhere near as many difficulties as does calculation for its own sake. Further, more specific remarks concerning calculation occur scattered through the latter part of this introduction and in the introductions to chapters of the World of Numbers strand.

As for "grading of achievement", CSMP takes the position that learning often stops when the purpose of a lesson or sequence of lessons is the mere mastery of skills. Every child should learn something from each situation; otherwise, it is not a fruitful situation and should be revised or eliminated. But to measure what or how much has been learned in any one situation is probably not possible. Learning is not a linear process. It occurs in quantum jumps that take place at times when you least expect them. It is possible to monitor the progress of students, and you all have your own ways of doing this, but who is to say at what precise time everyone in a class is supposed to have gained a particular skill or concept from his or her experiences? We comment further on this matter in the next section of this General Introduction.

A more substantive question is: How can we present situations rich in mathematical content before children have the verbal equipment and the computational skills to deal with them? It is presupposed by many people that children must be able to read and to calculate before they can tackle any mathematical situation other than trivial ones. Yet if one analyzes the types of thinking and the few basic concepts underlying elementary school mathematics, one finds that there is nothing esoteric or even difficult about these thinking modes or these concepts. What has been difficult for students in the past is the language in which these modes of thinking and these concepts are traditionally couched. For some reason, in our natural languages these modes of thinking are difficult to communicate precisely. That is why mathematicians devise languages of their own, languages that minimize the ambiguities of natural languages. Part of the problem in the past has been that elementary school children have found it difficult to deal with such tricky uses of language at a time when they do not yet have a firm command of their natural language.

On the other hand, since the underlying ideas and modes of thinking are inherently simple, there is no reason why very young children should not have the pleasure of such thinking and of exploring mathematical ideas. All that is needed are languages with which to express such ideas and to communicate the thinking that is generated by the ideas. These languages must be very simple, essentially nonverbal, and precise.

CSMP uses three such languages that fit the criteria and that have been highly successful in allowing students to do the kind of thinking that we described above. We call them the language of strings, the language of arrows, and the language of the Papy Minicomputer.

The first two are picture languages: brightly colored strings and dots enable precise descriptions of and the accurate recording of thinking about classifications of objects; colored arrows between pairs of dots stimulate thinking about relations between objects. For detailed descriptions and discussions of these two languages you are referred to the introduction to the Languages of Strings and Arrows strand.

The third language is based on a simple abacus called the "Papy Minicomputer". Using this language, one may represent all decimal numbers (positive or negative) in such a way as to encourage creative thinking about the nature and properties of those numbers. With the support provided by the Minicomputer, the children, even at an early age, have the opportunity to develop excellent mental arithmetic skills and to gain sophisticated insights into the world of numbers. The ability to perform mechanical computation is one of the incidental by-products of such an approach to work with numbers.

2. Program Description

a) Description of Materials

The CSMP Intermediate Grades curriculum is designed as a program for students in grades four, five, and six. There are six parts to the curriculum, each corresponding to one semester's work. This is Part IV of the six. It follows Part III of the CSMP Intermediate Grades Program.

In addition there are three Entry Modules, one for each grade level. As the name implies, each Entry Module is designed to facilitate transition into the CSMP Elementary School curriculum.

The materials of Part IV of CSMP Mathematics for the Intermediate Grades require some explanation. This collection of three volumes (six books) which are directed specifically to you, the teacher, is known as the "Teacher's Guide to CSMP Mathematics for the Intermediate Grades, Part IV". (By way of abbreviation, in what follows we shall refer to this collection simply as the "Teacher's Guide".) With the exception of this book, "General Introduction", each book of the Teacher's Guide addresses itself to a single area of the curriculum, and each of these areas is known as a "strand". The four content strands are:

- the World of Numbers - (N);
- the Languages of Strings and Arrows - (L);
- Geometry and Measurement - (G);
- Probability and Statistics - (P).

The six books comprising the "Teacher's Guide" are bound into three volumes, with a separation page inserted between the various books of a given volume.

This volume contains both this General Introduction as well as the Workbook (W) strand. One of the other two volumes contains the World of Numbers strand, and the other contains the Languages of Strings and Arrows stand, the Geometry and Measurement strand, and the Probability and Statistics strand. Each of these six books has its own table of contents and is paginated individually.

The content and Workbook strands have the following arrangement of material.

- A table of contents. This enables you to locate whatever you need in that particular strand.
- A suggested chronological order of the lessons in that strand. This provides any easy, quick reference uncluttered by extraneous information. It shows you in which order the lessons described in the strand will be taught if you follow the suggested schedule.
- An introduction to the strand. This provides you with information explaining why certain material finds its place in that strand, what are the overall goals of study in that strand, and how this material fits in with that which is described in other strands.
- Suggestions for the planning and conducting of lessons in that strand. These include lesson descriptions, suggestions on the use of worksheets and manipulative materials, general remarks relating to certain sequences of lessons, etc.

The suggested lessons are grouped in chapters, all the lessons described in the same chapter having some particular feature(s) in common. Because of this commonality of purpose, it is possible to precede each chapter by an introduction which indicates where the lessons in that chapter are going and, on occasion, gives a few more details concerning the mathematical underpinnings of the lessons described in the chapter than are contained in the lesson descriptions themselves.

Many of the suggested lessons require student worksheets for individual work.

Classroom sets of these worksheets are printed and packaged separately from this guide. However, an answer key for each of the worksheets is included in the guide. Worksheets are numbered consecutively within each strand and their answer keys can be found at the end of the lessons for which the worksheets are needed. For example, an answer key for Worksheet L11 is at the end of Section 2.4 of the Languages of Strings and Arrows strand since it is suggested that this worksheet be used with the lesson described in that section.

In order to accommodate the various ability levels in a given class, the worksheets generally are written at several levels of difficulty denoted by stars. "*" represents an easy worksheet, "**" represents an average level worksheet, and "***" and "****" represent difficult level worksheets. All students should begin with the "*" level worksheets for a given lesson or sequence of lessons and proceed as far as possible through the other levels. In classes containing the usual range of abilities, most of the students should succeed with the * worksheets; the ** worksheets should be appropriate for about two-thirds of the class; and approximately one-third of the students will find the *** and **** worksheets challenging and enjoyable to do. No one should ever be required to attempt all the worksheets provided for a given lesson. Each student should progress as far as his or her abilities allow.

There are various other items which are (or could be) used in the program. Among them are the following: various manipulative materials such as translators, rulers, grid board, and so on (these are described in more detail in the lessons in which they are used); several workbooks, storybooks, and story-workbooks (these are described in more detail in Sections 2 (b) and (e) of this General Introduction, and in the introduction to the Workbook strand); a book entitled "Math Play Therapy", which includes a collection of suggested small group activities that could be instrumental in stimulating an open and positive attitude toward mathematics on the part of those of your students who find the subject difficult. See the introduction to "Math Play Therapy" for suggestions as to its use in the program.

b) Role and Use of Workbooks

It is suggested that approximately one day each week be scheduled for students to do individual work in designated workbooks. The workbooks are designed to review ideas encountered in the content strands and to apply these ideas to new situations posed as stories. Workbooks are written with problems of varying levels of difficulty in order to accommodate differences in students' abilities and interests.

More detailed descriptions of the workbooks, their purposes, and answer keys are given in the Workbook strand. There you will also find suggestions for keeping records of student progress.

c) The Spiral Development

The learning process occurs through many interrelated experiences. But no single experience, particularly a 40 to 50 minute mathematics lesson, is an end in itself. The approach in the CSMP curriculum keeps this in mind. It is neither intended nor expected that every student will meet the full challenge of a particular situation in any given lesson. Furthermore, we find that it is most effective to vary the situations and strands from day to day rather than to continue with one type of situation or piece of content until so-called mastery has occurred. Thus, we suggest that the lessons be scheduled so that a different topic is studied each day of the week. In a typical week, for example, your class might work with numerical games one day, probability the next day, workbooks the third day, numerical relations the fourth day, and geometry the fifth day.

This spiral development, in which a student experiences each of several ideas a little at a time and then proceeds through increasing levels of sophistication as the situations become more challenging, reflects the fact that learning is not a linear process. Not only do different children learn at different times and different rates, but even on an individual level learning can occur in quantum leaps that take place at times when you least expect them. The spiral development gives each student a new chance to catch on to an idea at each turn of the spiral.

Of course, acceptance of the CSMP pedagogy requires a certain act of faith on your part. If you are accustomed to staying with a given topic until everybody (or nearly everybody) in the class knows it "cold", then you may be initially very reluctant to leave a lesson at the end of the time suggested, believing that several students "haven't got it yet". By keeping the spiral idea in mind, you should become more comfortable with these time schedules. Although it may appear to you that little learning is taking place in a sequence of lessons, you may be surprised that suddenly even your slowest students are understanding the ideas when they encounter them later in the spiral. Whereas with a mastery approach, presented in a purely individualized mode, slow students often spend most of the school year trying to "master" low level skills (with generally poor results), the spiral approach gives these students the opportunity to benefit from the richness of the situations presented throughout the curriculum. In this manner students are allowed to be involved in interesting and enjoyable situations which are modeled by important mathematical notions. They are not deprived of a variety of experiences just because they haven't mastered every bit and piece along the way. Therefore, in the long run, their interest in, understanding of, and even facility with mathematics has a much better chance to develop.

d) Evaluation

On the basis of the preceding discussion, it should come as no surprise, as stated earlier, that CSMP takes the position that learning often stops when the purpose of a lesson or sequence of lessons is the mere mastery of skills. Furthermore, we do not believe it is desirable or reasonable to attempt to assess student mastery of skills from day to day or even from week to week. As we said before, learning has a habit of occurring when you least expect it, so how does one determine when everyone in a class should have mastered a specific skill or attained understanding of a particular concept as a result of his or her experiences? We know of no method for measuring what or how much has been learned from any one situation except in the most trivial cases. On the other hand, teachers, parents, and even the students themselves can benefit from some continuous monitoring process that records involvement in activities and assesses progress through the program. Indeed, it is possible to monitor your students' progress without contradicting the statements made above; all of you have your own ways of doing this. Such a process is suggested in the introduction to the Workbook strand. Of course, you should also continue to use any monitoring devices with which you feel comfortable.

As far as traditional computational skills are concerned, the standardized test data collected during the extended pilot test stage of the development of this curriculum indicate that CSMP classes generally do better than average on the items which test "basic" skills.

About 75% of the lessons for the CSMP Intermediate Grades Program involve numerical concepts. The situations do not involve just the positive rational numbers (as is generally the case at this level), but introduce the students to many other citizens of the world of numbers at as early a stage as possible. You should not infer from this that your students are expected to have mastered the arithmetic of the whole numbers, the integers, the decimal numbers, and the rational numbers before they are ready for Part V of the CSMP Intermediate Grades Program. On the contrary, it means that the students will become more and more friendly and at ease with the various types of numbers and will develop their own methods for calculating with them long before some standard mechanical algorithms are introduced.

It is essential that the creative freedom of the student not be impaired by a premature emphasis on the standard algorithms for adding, subtracting, multiplying, and dividing. We believe that many of the present computational difficulties experienced by elementary and junior high school students were caused by their spending an inordinate amount of time trying to master the algorithms too early in their mathematical education. This shuts the students off from other experiences necessary for their mathematical growth. Students need plenty of time to explore the world of numbers. Mastering an algorithm should be the last step of this exploration, a sort of natural synthesis of previous experience. Furthermore, as electronic hand-calculators become available to nearly everyone, algorithmic mastery as a prerequisite to other numerical experiences becomes less important.

We strongly urge you to follow the suggested schedule of lessons, teaching the algorithms as they come, but not giving them more emphasis than they deserve. Skill in mental arithmetic and estimation, however, is extremely important in our opinion and should be promoted at every opportunity.

While CSMP disagrees with the philosophy underlying standardized testing, we realize that students as well as teachers are under extreme pressure to perform in certain ways with regard to such tests. If your students or their parents express concern about the apparent lack of emphasis on mastery of standard algorithms (such as long division by three or four digit numbers), they should be reassured that CSMP students have a definite advantage when it comes to applying mathematics to novel situations, strategic thinking, organizational ability, and understanding of the methods of mathematics. Because CSMP feels it is wrong for such tests to dictate emphasis on the mastery of standard algorithms and that drill for algorithmic mastery for its own sake defeats CSMP's purposes, you will find that even after algorithms have been introduced into the CSMP curriculum the role they play will be as tools for doing other interesting things. Therefore, if you feel pressured by the tests or otherwise to put more emphasis on drill activities, then we suggest you use any supplementary materials that have proved successful in the past to provide this extra practice. We leave scheduling choices for these kinds of activities to your judgement.

e) Role and Use of Storybooks and Story-Workbooks

Among the instructional materials for the CSMP Elementary School curriculum is a collection of mathematical storybooks, STORIES BY FREDERIQUE. This collection consists of 24 stories for young people, presented in colorful newsprint booklets. They are intended to appeal to youngsters in the 5-14 age bracket. The stories are completely independent of each other and need little or no external explanation. They may be used for individual or collective reading, at home or at school.

Each of the stories is portrayed by the mathematical languages of dots, strings, and arrows which reinforce the ideas of the story. Three of the stories (TWO BY TWO, THE WEIRD STORY OF 24, and SUMMER SCHOOL IN THE OLD DAYS) also use the language of the Minicomputer. The amazing versatility of these nonverbal languages is exploited to the full.

Young readers can become personally involved in the stories; there are questions to answer, and pictures to relate to the stories, and new adventures to invent for the characters in the stories. The reader discovers that emotions and aesthetic senses can be involved in the world of mathematics just as in the world of people.

Twelve of the stories are intended for very young children (5 to 8 years of age). Here the focus is on the anecdotal aspect of the story. One of the goals of these twelve books is to familiarize children with several nonverbal mathematical languages and to leave them with the impression that these languages are as natural and useful as the usual language.

Nine of the remaining stories concern numbers and their relationship with the world. These particular stories are about a boy and some of his friends who just happen to be numbers. The personification of numbers introduces the "games" of mathematics into one of the most important domains of childhood -- the world of fantasy. Free from the usual constraints so often placed upon them, the numbers invent marvelous games and dance to the tunes of vividly colored arrows. These games are never frivolous. There are rules to be followed and as a result, orderly patterns emerge as the numbers play or dance.

Of the remaining three books, two are detective stories introducing some interesting mathematical problems, and one emphasizes the various and very different aspects of a simple straight line.

The titles of the books in the collection and the intended age range for each story are:

Ages 5 to 8

THE PLAYFUL NUMBERS
81 ROSES
I AM A VERY HAPPY BOY
ONE OUT OF SEVEN
THE HAPPY PUPPET
TWO BY TWO
THE LITTLE DREAMER
THE LITTLE DONKEY
THE MAGIC BOX
THE BABY IS BORN
THE WEIRD STORY OF 24
THE OLD SHOEMAKER

Ages 8 to 12

WHERE'S MY NOSE?
SINGING FRIENDS
DANCING FRIENDS
THE LIVING LINES
I AM NOT MY NAME
NABU WINS AN AWARD
THE SQUARE TRAP
SUMMER SCHOOL IN THE OLD DAYS

Ages 10 to 14

THE HIDDEN TREASURE
A VERY STRANGE NEIGHBORHOOD
ELECTION IN THE NUMBER WORLD
A VALENTINE MYSTERY

In the classroom set of materials for Part IV of the Intermediate Grades Program, two copies each of the following storybooks are provided: TWO BY TWO, WHERE'S MY NOSE?, SINGING FRIENDS, DANCING FRIENDS, THE LIVING LINES, I AM NOT MY NAME, NABU WINS AN AWARD, and THE SQUARE TRAP. They should be put in the class library for students to read whenever they like. These are books that were in the third grade program and in previous parts of the Intermediate Grades Program and that students may enjoy reading from time to time. If Part IV is being used in the second semester, following Part III in the first semester, these storybooks are not included with Part IV materials since they were provided with Part III materials.

Two of the storybooks, THE HIDDEN TREASURE and A VALENTINE MYSTERY, are the basis for lessons in the Workbook strand. The classroom set of materials for Part IV contains 15 copies of each of these storybooks; however, you may wish to order additional copies so that every student may have a copy.

Besides the collection, STORIES BY FREDERIQUE, another collection, THE CSMP LIBRARY, has been developed as part of the instructional materials for the CSMP Elementary School Program. This is a series of story-workbooks which combines the spirit of the storybooks and the workbooks into a single integrated experience for the children, engaging them in reading about and doing mathematics in stimulating, challenging story situations. The current list of story-workbooks in the collection is:

Ages 8 to 11	Ages 9 to 14
ROLLERSKATING 37	SUMMER CAMP
TO PICTURE	HALLOWEEN PUZZLES
A SHORT STORY ABOUT 2 FRIENDS	1,000's DREAM
A SHORT STORY ABOUT 3 FRIENDS	A STRANGE COUNTRY
THE ISLAND OF TAM - TAM	NOT TOO CLOSE
THE TALKATIVE NUMBERS	WHO ARE WE?
	CLINTON STREET
	A BOOK ABOUT ME
	SEVEN SECRET NUMBERS
	SHUNDA'S NEWSSTAND

f) Role and Use of Hand-Calculators

The electronic hand-calculator, if used creatively, can have a remarkably beneficial effect on the students' interest in and understanding of arithmetic, and its use can be a contributing factor in the improvement of their capabilities in that area. The reasons for this are many and varied, but among them are certainly the following:

- By removing the burden of mechanical computation from the children's shoulders, the calculator frees them to concentrate on the thinking that is a necessary part of the solution to any meaningful problem.

- The instant feedback provided by the calculator enables the children to indulge in making and testing conjectures, an area into which they would not otherwise venture because the quantity of computation involved would be too forbidding.

This pedagogical tool is particularly useful in the introduction of new numerical notions such as negative integers, decimal numbers, multiples of a given whole number, and so on, because through suitably chosen, rich situations these concepts can be made to arise in a natural way.

A classroom set of battery powered hand-calculators is recommended for Part IV. Many different kinds of hand-calculators are available from CSMP as a distributor for National Semi-Conductor and Texas Instruments. Any of the products of these two companies may be purchased at discounted prices.

If it is not possible to supply all students with hand-calculators, encourage the students to bring calculators from home. The hand-calculators that are available from CSMP have algebraic logic and function keys which are permanently in a constant mode. You should check to be sure that calculators that you supply or that are brought in by students also have these characteristics.

- Algebraic logic. This means that the calculator responds to instructions given in the order in which they are usually written. To check that this is the case, press $\boxed{2}\,\boxed{+}\,\boxed{3}\,\boxed{=}$. If you have "5" on the display, then all is well.
- Constant mode. Some calculators have a switch so that the calculator can be set either in a constant mode or in a chain mode. If your calculator lacks a switch, it is quite likely that the calculator has an automatic constant. For calculators with an automatic constant, the first number entered during multiplication is usually the constant, while the second number entered during the other three operations (addition, subtraction, division) is usually the constant.

To check your calculator for the constant mode operation of $\boxed{+}$ is a simple procedure. Press $\boxed{0}\ \boxed{+}\ \boxed{2}\ \boxed{=}\ \boxed{=}\ \boxed{=}$ and so on. If the number on the display is increasing by two at each depression of $\boxed{=}$, then $\boxed{+}$ is operating in a constant mode. Check the constant mode operation of $\boxed{-}$ in a similar fashion.

To check the constant mode operation of $\boxed{x}$ (where the first number entered will be the constant) press $\boxed{2}\ \boxed{x}\ \boxed{1}\ \boxed{=}\ \boxed{=}\ \boxed{=}$ and so on. If the number on the display doubles at each depression of $\boxed{=}$, then $\boxed{x}$ is operating in a constant mode.

While it would be possible to use calculators without these characteristics in the lessons, the adjustments that would be required in the instructions make the calculators unsuitable.

The hand-calculator relations $\boxed{+}\ \boxed{n}\ \boxed{=} \ldots$ and $\boxed{+}\ \boxed{=} \ldots$ are explored quite extensively in several lessons. Even if your calculators have algebraic logic and the constant mode, it may be necessary for you to check to see if your calculators operate in the way that is described in the lessons.

If you find that your calculators do not operate in this manner, then try letting

$\boxed{+}\ \boxed{n}\ \boxed{=} \ldots$ mean

$\boxed{+}\ \boxed{n}\ \boxed{=}\ \boxed{+}\ \boxed{n}\ \boxed{=}\ \boxed{+}\ \boxed{n}\ \boxed{=} \ldots$

and

$\boxed{+}\ \boxed{=} \ldots$ mean

$\boxed{n}\ \boxed{+}\ \boxed{0}\ \boxed{=}\ \boxed{+}\ \boxed{n}\ \boxed{=}\ \boxed{+}\ \boxed{n}\ \boxed{=} \ldots$

where n is some starting number.

Furthermore, with some calculators there is a correspondence between a multiple of a number and the number of times $\boxed{=}$ is pressed in the $\boxed{+}$ $\boxed{=}$ ... relation. For example,

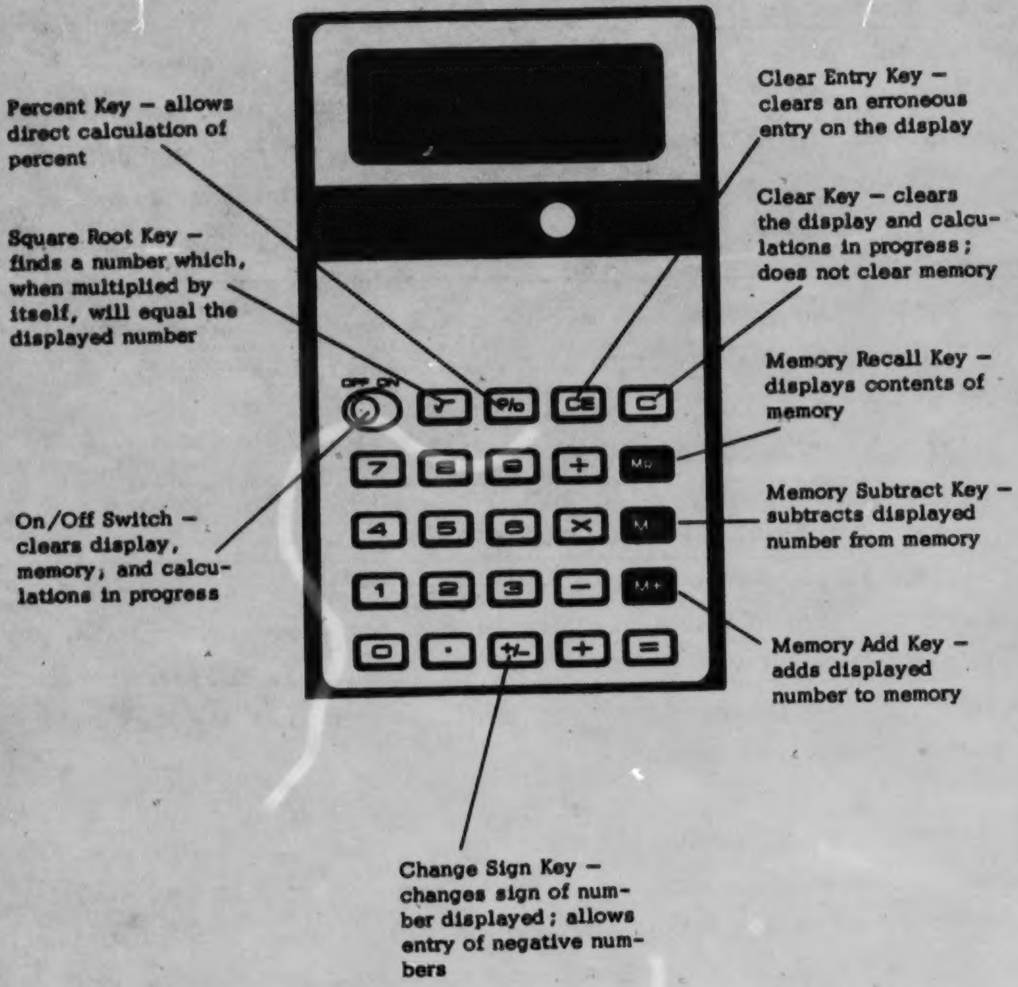
> Starting with "4" on the display and pressing $\boxed{+}$ $\boxed{=}$ yields 4 (= 1 x 4). Pressing $\boxed{=}$ again yields 8 (= 2 x 4); pressing $\boxed{=}$ again yields 12 (= 3 x 4); pressing $\boxed{=}$ again yields 16 (= 4 x 4); pressing $\boxed{=}$ again yields 20 (= 5 x 4); and so on. Notice that when $\boxed{=}$ has been pressed 5 times, the fifth positive multiple of 4 (i.e., 20) is displayed.

Frequently, calculators that do not behave in the manner described above will differ by one less (i.e., the fifth positive multiple of 4 appears after pressing $\boxed{=}$ four times).

The calculators used in the descriptions of the lessons of Chapter 7 HAND-CALCULATOR PUZZLES are this latter kind. Therefore, there may be some discrepancy between solutions your students find and the ones presented in those lesson descriptions. However, the differences should be minor and you should be able to adjust the lesson description to fit the type of calculators your students are using.

In Part IV you and your class will have several opportunities to explore the use of "special keys" that appear on many hand-calculators (see Sections N3.7 and N3.11 and Chapter 7 of the N strand). The illustration on the following page gives a brief description of some "special" keys that might be on your hand-calculators.

Prior to the presentation of the hand-calculator lessons, you should familiarize yourself with these keys.


Percent Key — allows direct calculation of percent
Square Root Key — finds a number which, when multiplied by itself, will equal the displayed number
On/Off Switch — clears display, memory, and calculations in progress
Clear Entry Key — clears an erroneous entry on the display
Clear Key — clears the display and calculations in progress; does not clear memory
Memory Recall Key — displays contents of memory
Memory Subtract Key — subtracts displayed number from memory
Memory Add Key — adds displayed number to memory
Change Sign Key — changes sign of number displayed; allows entry of negative numbers
OFF ON
%
CE
C
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8
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3. Guidelines for Organizing the Program

The design and content of the lessons in Part IV affords you the utmost flexibility in teaching the program successfully. It is a truism to say that your students this year are very different, both individually and as a class, from the students you taught last year, and indeed from other students in any other CSMP class. And your own teaching style and interests are probably very different from those of your colleagues. It is difficult for those of us who develop curricula to guess correctly as to which topics will be very easy for your students and which lessons will require more instructional time. You are the only one who will be able to judge that as you begin using the materials. This program is not meant to be a static sequence of exercises to be followed rigidly; the schedule and design of lessons in this guide should be regarded as suggestions; they simply reflect our own experience teaching the materials. You and your students will make the program dynamic as together you explore the mathematical ideas and follow your own interests and inclinations.

a) Use of Schedules

If this is the first time you are teaching a CSMP class and you would like specific suggestions for organizing the lessons, we offer some guidelines and a suggested schedule of lessons. Please remember that our suggested sequence is but one of many that could have been assembled. You may wish to follow it at the beginning of the school year and then deviate from it as the needs of your students may dictate.

The master schedule included at the end of this section reflects our suggestions for a hypothetical fifth grade class of average ability, and it illustrates some important ideas about organizing the program. You will notice that several different strands are scheduled each week and that even within a strand the topics are interspersed. This was done to carry out the idea of a spiral curriculum. If you organize your own schedule, it is best to follow a similar pattern rather than to spend two or three full weeks, for example, "covering" one particular topic such as composition of relations or decimal numbers. In a usual five day week, two days should be devoted to the World of Numbers strand, one day to the Workbook strand, and other two days should be shared by the P, G and L strands.

In the schedule, a block represents one full period of approximately 45-50 minutes duration. It may very well be, however, that you and your class will spend more than one period on some lessons. Exercise your own professional judgment in deciding when to stop one particular lesson and when to take it up again. Just be sure that you do not insist that every single child understands every single aspect of a lesson before you go on. It is better to stop a lesson at a point when student interest is high and a problem remains unsolved than to teach so methodically that everyone in the class understands everything, but is completely bored.

Inside each block, the lesson of the day is designated in several ways. First, there is listed a number which indicates its suggested chronological order among the lessons of a strand. Hence, "5N" indicates that the lesson in question is suggested as the fifth lesson in the N (World of Numbers) strand. This identifying number also appears in the upper outside corner of each page in the description of that lesson, thus enabling a lesson to be located simply by flipping through the pages of the strand book. Second, you are given the section number and title of the lesson. Third, you are told on which page of the strand book the description of the lesson in question begins.

Please bear in mind that the model schedule was designed for a hypothetical class with the usual range of abilities. In order to be successful with the program, you should adjust the schedule to the ability, maturity, and interests of your class. For example, you may wish to extend a lesson or a workbook for an additional period beyond those scheduled.

a) Use of Adjustment Days

From time to time an adjustment day appears in the schedule. Since we have intended that Part IV would be the mathematics curriculum for one semester of the Intermediate Grades Program and a usual school semester is 18 weeks, the schedule indicates 18 weeks of lessons. However, it is also usual for there to be days when no mathematics lesson is taught; e.g., parent-teacher conference days, field trip days, snow days, and so on. These adjustment days should account for such non-class days, and should also give you opportunities to do many kinds of things. You could use an adjustment day to review a lesson you feel needs review, or to complete an unfinished lesson, or to extend some interesting ideas from completed lessons. You could also use an adjustment day to give your students time to become more familiar with the manipulative materials in the program, such as the translators or the cutout maps of a cube or the storybooks. Your students may especially enjoy playing games such as Minicomputer Golf, Minicomputer Tug-of-War, Guess My Rule, or the String Game, and an adjustment day could give them another opportunity to play a game. You may wish to use an adjustment day to do some mental arithmetic exercises, or to teach a topic in mathematics you think would be especially interesting or useful to your class.

SUGGESTED SCHEDULE OF LESSONS

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART IV

WEEK	The World of Numbers	Languages of Strings and Arrows
1	**1N** 1.1 Minicomputer Review N p. 3	**1L** 1.1 String Game Analysis #1 L p. 3
2	**3N** 2.1 Who is Paf? N p. 51	**2L** 1.2 Counting Problems #1 L p. 12
3	**5N** 4.1 Nabu's Abacus #1 N p. 207	**3L** 2.1 Images #1 L p. 59
4	**7N** 3.2 Newtonian Quiz N p. 120	**4L** 3.1 Map Distance #1 L p. 123
5	**8N** 3.3 Multiplication of Rational Numbers #1 N p. 129	**5L** 3.2 Map Distance #2 L p. 131
6	**10N** 2.2 Who is Crick? N p. 59	**6L** 1.3 String Game Analysis #2 L p. 20
7	**12N** 1.4 Multiplication #2 N p. 28	**7L** 2.2 Images #2 L p. 77
8	**14N** 6.1 Composition #1 N p. 287	**8L** 3.3 Parental Distance L p. 145
9	**16N** 4.3 Nabu's Abacus #2 N p. 236	ADJUSTMENT DAY

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART IV

Workbooks	The World of Numbers	Geometry and Measurement Probability and Statistics
1W A Selection of Problems #1 W p. 2	**2N** 1.2 Multiplication #1 N p. 12	ADJUSTMENT DAY
2W A Selection of Problems #1 W p. 4	**4N** 3.1 Rational Numbers and Multiplication Functions N p. 111	**1G** 1.1 Cubes and Volume #1 G p. 3
3W A Valentine Mystery W p. 15	**6N** 5.1 Algorithm Puzzles N p. 264	**2G** 1.2 Cubes and Volume #2 G p. 10
4W A Valentine Mystery W p. 22	ADJUSTMENT DAY	**3G** 2.1 Purchase Vector Space #1 G p. 42
5W A Selection of Problems #2 W p. 30	**9N** 4.2 Binary Code #1 N p. 223	**4G** 2.2 Purchase Vector Space #2 G p. 50
6W A Selection of Problems #2 W p. 30	**11N** 1.3 Guess My Rule Games N p. 21	**5G** 2.3 Purchase Vector Space #3 G p. 54
ADJUSTMENT DAY	**13N** 5.2 Division Algorithm #1 N p. 270	**6G** 1.3 Mapping a Cube #1 G p. 19
7W A Selection of Problems #3 W p. 40	**15N** 3.4 Decimals on the Number Line N p. 138	**7G** 1.4 Mapping a Cube #2 G p. 26
8W A Selection of Problems #3 W p. 43	**17N** 3.5 Sasquatch #1 N p. 146	**1P** 1.1 Random Number Stories #1 P p. 3

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART IV

WEEK	The World of Numbers	Languages of Strings and Arrows
10	**18N** 7.1 Hand-Calculator Puzzles #1 N p. 313	**9 L** 2.3 Guess My Rule #1 L p. 93
11	**20N** 4.4 Binary Code #2 N p. 250	ADJUSTMENT DAY
12	**22N** 3.7 Fractions and Decimal Names #1 N p. 166	**10L** 1.4 Counting Problems #2 L p. 34
13	**24N** 3.8 Addition of Rational Numbers #1 N p. 172	**11L** 2.4 Images #3 L p. 101
14	**26N** 5.3 Division Algorithm #2 N p. 277	ADJUSTMENT DAY
15	**28N** 2.5 Greatest Common Divisor & Least Common Multiple Relations #2 N p. 88	**12L** 2.5 Guess My Rule #2 L p. 112
16	**30N** 3.9 Multiplication of Rational Numbers #2 N p. 182	**13L** 1.5 String Game Analysis #3 L p. 45
17	**32N** 3.11 Addition of Rational Numbers #2 N p. 198	**14L** 3.4 Prime Factor Distance #1 L p. 155
18	**34N** 2.6 The Relation $=_3$ N p. 96	**15L** 3.5 Prime Factor Distance #2 L p. 168

MASTER SUGGESTED SCHEDULE OF LESSONS FOR PART IV

Workbooks	The World of Numbers	Geometry and Measurement Probability and Statistics
9W The Hidden Treasure W p. 53	**19N** 2.3 Greatest Common Divisor & Least Common Multiple Relations #1 N p. 66	**2P** 1.2 Random Number Stories #2 P p. 14
10W The Hidden Treasure W p. 63	**21N** 3.6 Sasquatch #2 N p. 156	**8G** 2.4 Purchase Vector Space #4 G p. 63
11W A Selection of Problems #4 W p. 74	**23N** 1.5 Sums of Sequences of Whole Numbers N p. 37	**9G** 1.5 Surface Area G p. 31
12W A Selection of Problems #4 W p. 74	**25N** 2.4 Who is Clip? N p. 78	**10G** 2.5 Purchase Vector Space #5 G p. 67
13W A Selection of Problems #5 W p. 86	**27N** 7.2 Hand-Calculator Puzzles #2 N p. 321	**3P** 2.1 Probability Games #1 P p. 37
14W A Selection of Problems #5 W p. 86	**29N** 6.2 Composition #2 N p. 299	**4P** 2.2 Probability Games #2 P p. 55
ADJUSTMENT DAY	**31N** 3.10 Fractions and Decimal Names #2 N p. 192	**11G** 2.6 Purchase Vector Space #6 G p. 71
15W A Selection of Problems #6 W p. 96	**33N** 7.3 Hand-Calculator Puzzles #3 N p. 329	**5P** 2.3 Probability Games #3 P p. 72
16W A Selection of Problems #6 W p. 98	ADJUSTMENT DAY	**6P** 1.3 Random Number Stories #3 P p. 22

4. Commonly Asked Questions about the CSMP Curriculum - and Suggested Answers.

As you teach the CSMP curriculum (or even as you prepare to teach it) there will surely be some basic questions that come to your mind. They are the kind of questions that not only you as a teacher, but also parents and supervisors, will ask about any curriculum that appears to be different from the usual program.

The following questions and answers are listed as an aid to you in understanding our curriculum and in responding to those who request information about it.

1. Q: FOR WHAT SORT OF STUDENT IS THE CSMP ELEMENTARY CURRICULUM DESIGNED?

   A: It is designed for all children in the schools. Each learning situation is designed to involve all students at their own levels in such a way that the slowest can succeed in dealing with some aspects of the situation that challenge them. In fact, it has been found that such a "pedagogy of situations" (see Question 2) is particularly effective with the so-called "slow" learners, and recent research suggests it might be effective also with educable mentally handicapped children.

2. Q: BRIEFLY, HOW DOES CSMP DIFFER FROM A TRADITIONAL MATHEMATICS CURRICULUM?

   A: The core content of CSMP is not revolutionary. The program aims to inculcate notions and facts which have always been considered important in mathematics. The major differences are in method. Traditional teaching takes pride in disciplining the student to be submitted to the usual algorithms. CSMP students are involved as well in many other real mathematical activities, with understanding at their level.

For that purpose CSMP uses new teaching aids, the nonverbal languages of strings and arrows and of the Papy Minicomputer, which are themselves both pure and applied mathematics. These mathematical objects (graphs, arrows, etc.) are used as aids rather than as ends in themselves.

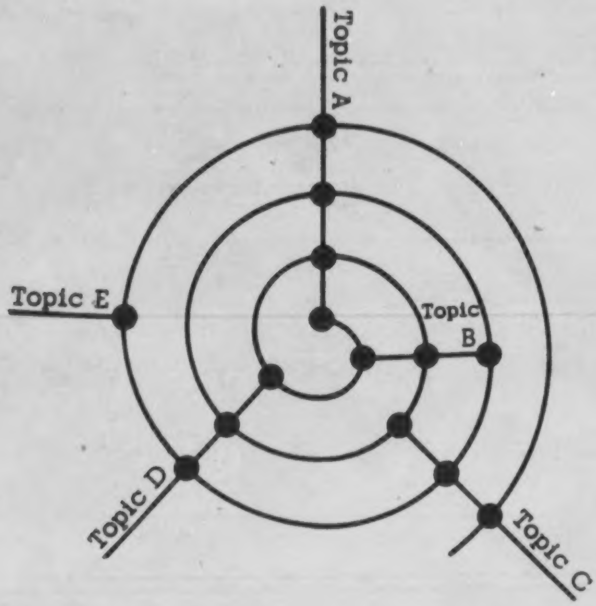
A pedagogy of situations is used to permit children to participate with creative freedom in the learning process. The students are faced with situations and are led by their reactions to important ideas in mathematics. Thereby, students learn to mathematize situations, a process which is of fundamental importance whenever mathematics is applied.

CSMP also makes use of a spiral approach (see Question 3) rather than the mastery approach found in most mathematics curricula.

3. Q: WHAT IS THE SPIRAL APPROACH? HOW DOES IT WORK?

A: The learning process occurs through many interrelated experiences. But no single experience, particularly a 40 to 50 minute mathematics lesson, is an end in itself. The CSMP curriculum was developed with this mind. It is neither intended nor expected that each child will meet the full challenge of a particular situation in any given lesson. Furthermore, we have found that it is most effective to vary the situations and topics from day to day rather than to continue with one type of situation or piece of content until so-called mastery has occurred. Children at this age do not seem to enjoy staying on the same topic for a very long time but prefer to be involved in a variety of situations. Thus, the lessons are scheduled so that different topics will be studied during each week and similar situations appear again and again at different times and levels in a spiral development.

If a spiral curriculum seems strange at first, consider the following illustration: Imagine an academic subject that could be divided into five subtopics. Then imagine a student's progress being represented by a spiral, starting at the center and traveling outwards in the direction of increasing degrees of maturity and sophistication. In the diagram, an intersection point between the spiral and a topic line indicates a lesson on that topic.



The student is first introduced to Topic A, then to B, and to D, followed by a slightly more sophisticated lesson on A, and so on. Topics C and E are not introduced until a little later in the spiral, while Topic B may be of a more terminal nature and is considered completed after a few lessons.

This spiral development, in which a child experiences each of several ideas a little at a time and then proceeds through increasing levels of sophistication as the situations become more challenging, is wholly consistent with CSMP's pedagogy of situations.

Acceptance of the CSMP pedagogy requires a certain act of faith on your part. If you have been accustomed to staying with a given lesson until everybody (or nearly everybody) in the class knows it "cold", then you may be initially very reluctant to leave a lesson at the end of the time suggested, believing that several children "haven't got it yet". By keeping the spiral idea in mind, you may become more comfortable with these time schedules. Although it may appear to you that no learning is taking place in a sequence of lessons, you may be surprised that suddenly even your slower students are understanding the ideas when they encounter them later in the spiral. Whereas with a mastery approach, presented in a purely individualized mode, slow students often linger most of the school year trying to "master" low level skills (with generally poor results), the spiral approach gives these students the opportunity to benefit from the richness of the situations presented throughout the curriculum. In this manner, children are allowed to become involved in interesting and enjoyable situations which are modeled by important mathematical notions. They are not deprived of a variety of experiences just because they haven't mastered every bit and piece along the way.

4. Q: ISN'T THIS THE "NEW MATH"?

A: There is a current popular belief that all school mathematics today is either "new math" or "old math". Neither of these labels is well defined. "New math" usually denotes the programs that heavily emphasize the teaching of sets; introduce technical mathematical language for concepts such as "commutativity", "associativity", "distributivity", and so on; and that teach "concepts" before "skills". The "old math" refers to programs of the "good old days": memorization of the multiplication tables and complete emphasis on paper and pencil drill in calculations.

The CSMP curriculum is definitely not the "new math" nor the "old math" by any definition. It uses no technical language, yet achieves the understandings sought by the "new math". It uses no repetitive drills, yet elicits the ability to calculate stressed by the "old math".

But these achievements are just by-products of a more pervasive goal: to help children use their curiosity, their intuition, their powers of reasoning to formulate and think about interesting problems and situations.

5. Q: IS THE STUDENT REALLY STUDYING MATHEMATICS IN THE CSMP CURRICULUM?

A: Yes. The subject matter of mathematics involves much more than calculation with numbers, and the methods of mathematics include more than the paper and pencil algorithms. CSMP believes that the number systems are very important and that it is essential to introduce students to the world of numbers. However, the intelligibility of that world is not simply a matter of computation. The CSMP curriculum is filled with experiences that directly and indirectly relate to the wide world of numbers, that support important internal relations in the world of numbers, and that encourage logical reasoning. The deep understanding of mathematics is not always verbal but is very often pictortorial and CSMP tries to respect that aspect.

6. Q: WHY USE THE LANGUAGES OF STRINGS, ARROWS, AND THE PAPY MINICOMPUTER?

A: It is surprising that the language of strings was not used earlier and more extensively in the teaching of mathematics. Mathematicians have used strings for a very long time and they support notions so fundamental and natural that they go back to Aristotle. The scope of the string diagrams is broader than mathematics, the string diagram being a marvelous instrument for logical thinking. String diagrams permit the understanding of a situation at a glance.

Arrow diagrams are probably the most important teaching aid in CSMP. They are used everywhere in our daily lives: signals, advertisements, directions, and so on. Why not use them in teaching mathematics?

Arrows are a pictorial way to represent relations. Relations have always been important in mathematics, but in the past seemed to be something rather abstract, elusive, and mysterious. With arrows they become vivid, concrete, and easy to understand.

The Minicomputer allows very young children to become familiar with numbers and operations on them (especially large numbers and the more difficult calculations) before they are able to work with them on paper. Later, and more importantly, the Minicomputer is used as a vehicle for posing situations involving mental arithmetic and for examining the workings of the arithmetic operations. It provides the experience necessary so that rather than memorizing an algorithm and applying it to situations, the situations give rise to the algorithm.

7. Q: IF THE STUDENTS USE THE MINICOMPUTER, WILL THEY BE ABLE TO CALCULATE WITHOUT IT?

A: This question suggest that the only purpose of the Minicomputer in the CSMP curriculum is to perform calculations. To be sure, the Minicomputer does enable very young children to deal with "large" numbers and perform relatively difficult calculations long before they can write the paper and pencil algorithms for the calculations. But very soon these experiences with the Minicomputer suggest how the paper and pencil algorithms come about, and from that moment the Minicomputer is no longer regarded as a calculating tool. Instead, for the remainder of the curriculum, the Minicomputer assumes its primary roles: as a source of very interesting problems and games, as an effective vehicle for stimulating mental arithmetic, as a teaching device for introducing more advanced topics such as the negative numbers and the decimal numbers, and, in general, as an alternate representation of numbers.

8. Q: WHAT IS THE CSMP POSITION ON HAND-CALCULATORS AND WHAT ROLE DO THEY PLAY IN THE PROGRAM?

A: Hand-calculators should be a part of any mathematics curriculum. As more and more children have hand-calculators, these tools will become incorporated in the elementary and secondary curricula in a natural way with uses very similar to those of the Minicomputer. In some cases, they may be used as a tool for mechanical computation where the difficulty of the calculation might turn an otherwise interesting problem into an exercise in drudgery. In other cases, they may be used as the context for posing situations leading to the discovery of important mathematical properties or providing opportunities for interesting exercises in mental arithmetic.

Hand-calculators are of particular benefit to "slow" learners. Many slow learners have mechanical difficulties rather than problems in thinking about the situation. The hand-calcualtor alleviates this problem.

Some lessons in the CSMP curriculum require only two calculators, while others suggest that each student have one. Sometimes lessons of this latter sort can be modified so that students may share hand-calculators or may simply think about how a hand-calculator works. For some lessons, however, every student should be provided with a hand-calculator. In either situation, students with access to hand-calculators should always be encouraged to bring them to class.

9. Q: WHY IS THERE A LACK OF DRILL AND LITTLE EMPHASIS ON "SKILLS"?

A: CSMP believes that training for rote skill should never precede understanding. The avowed purpose of drill in arithmetic is to acquire the "skills", that is, the skills of calculation. Certain numerical skills are essential, but, unfortunately for school children, the usual drill techniques for acquiring them are dull, stultifying, and sometimes even counterproductive. The CSMP curriculum appears to place little emphasis on "skills" because it does not resort to drill. But as a matter of fact, the essential skills of both mental and written calculation are learned almost inadvertantly in response to the many situations where the ability to calculate is needed in order to respond to the situation: to win the number game, to identify the secret number of the detective story, or to build a number road. Calculation is always considered as a tool for doing interesting things, rather than a chore done for its own sake. See also the comments on Evaluation, Section 2 (d) of the General Introduction.

10. Q: HOW DO STUDENTS LEARN TO USE MATHEMATICS IN PRACTICAL SITUATIONS?

A: For students of elementary school age, a situation that intrigues them and catches their imagination or their curiosity will certainly motivate them to learn whatever is necessary in order to deal with the situation. But situations that adults consider "practical" very often are outside the realm of interest of school children and do not provide motivation for learning. CSMP has found that students will indeed learn to use mathematics in situations they consider worthy of their attention. As they approach adulthood they will soon consider "practical" situations to be most worthy of their attention, but in the meanwhile, they are more interested in games, fantasy, puzzles, stories, and so on.

11. Q: WHY ARE WE TEACHING PROBABILITY?

A: Next to arithmetic, probability has the most direct application to life in such areas as economics, meteorology, data processing, insurance, most of the natural sciences, and most of the social sciences. Probability differs from many areas of mathematics in that it is applied directly. One does not need to combine knowledge of probability with any deep theoretical knowledge in order to apply it to "real world" problems. One encounters statistics nearly every day of one's life, and the better developed one's intuitive sense of probabilistic situations is, the less likely one is be "duped" by statistics.

12. Q: HOW ARE TELLING TIME, MONEY, AND THE METRIC SYSTEM TREATED?

A: Telling time is not included in the CSMP curriculum. It is our belief that telling time is more naturally a part of a language program than of a mathematics program, although it is frequently included in the mathematics curriculum. Since there are many excellent materials available to help you teach telling time, we urge you to teach this topic using those methods and materials you particularly like or that have proved successful for you in the past.

The topic of money is first introduced in the CSMP curriculum in lessons involving combinations of coins. It is treated again in the Upper Primary Grades and the Intermediate Grades as a support for the teaching of decimal numbers. Although money is not treated as a separate topic, these situations provide considerable experience with our monetary system. For students who need more exposure to this topic, teachers should feel free to develop supplementary materials in order to provide extra practice.

Metric units are used whenever there is an occasion to measure in standard units. Activities involving the metric system are begun in the First Grade Program and continue throughout the Geometry and Measurement strand of the Upper Primary Grades Program and the Intermediate Grades Program. We recommend that you do not ask the students to convert from the English to the metric system or vice versa (inches to centimeters, yards to meters, Celsius to Fahrenheit, grams to ounces, and so on). Since the United States has not yet "gone metric", your students will still need to be familiar with the English system. We suggest that you use whatever materials and methods you have found successful in the past for teaching this topic, but we repeat our recommendation that you not ask the students to convert from one system to the other.

13. Q: WHAT IS THE ROLE OF STORYBOOKS AND STORY-WORKBOOKS IN THE PROGRAM?

A: The storybooks in the collection STORIES BY FREDERIQUE include 24 stories for children, presented in colorful newsprint booklets. They are intended to appeal to children in the 5-14 age bracket. The stories are completely independent of each other and need little or no external explanation. They may be used for individual or collective reading, at home or at school. The story-workbooks, as the name implies, are a cross between storybook and workbook. They present more problems for the student to solve then do the storybooks, but within a story setting. The story-workbooks comprise the collection called "The CSMP Library". Each of the stories is portrayed by the mathematical languages of dots, arrows, and strings which reinforce the ideas of the stories. A young reader can become personally involved in the stories; there are questions to answer, and pictures to relate to the stories, and new adventures to invent for the characters in the stories. The reader discovers that emotions and aesthetic senses can be involved in the world of mathematics just as in the world of people.

Among the goals of the various storybooks and story-workbooks are the following: to familiarize the students with nonverbal mathematical languages and to leave them with the impression that these languages are as natural and useful as the usual language; to introduce some mathematical games; to present detective stories which introduce some interesting mathematical problems; to provide reinforcement for all students and especially for the slower learners.

14. Q: WHAT PROVISION IS MADE FOR INDIVIDUALIZATION IN THE CSMP CURRICULUM?

A: In many of the lessons there are worksheets to be done individually by the students. Sometimes the worksheets are done at the beginning of a lesson, followed by a full group discussion of them. But ordinarily a topic is first developed in a full group setting, followed by individual student work on related worksheets. If "individualization" means a precedure for allowing students to work alone at their individual rates and levels, then the worksheets (and also the workbooks, discussed below) do provide for (partial) individualization. A series of worksheets for a given lesson offers a whole spectrum of levels from very simple to quite challenging. Students all start with the first worksheet (one-star) and proceed at their own pace and levels as far as they are capable. The spiral approach accommodates this kind of individual work in the same manner as the full group work; that is, no worksheet is a prerequisite for the content of any succeeding lesson.

Twelve lessons during the semester are devoted to the six IG-IV workbooks. Each workbook is also designed to move through a spectrum of levels of difficulty. We believe that the worksheets and workbooks realize the best sense of "individualization".

15. Q: WHAT PROVISIONS ARE MADE FOR THE VARIETY OF ABILITIES IN A USUAL CLASS?

A: Every lesson includes something for all students; there are opportunities for the slowest students to participate in a meaningful way, and there are challenges for the brightest students. This means that you should not exclude any students from a full group lesson. At the same time, you should not expect every student to understand everything during the course of the lesson. Furthermore, with the spiral development many situations are open-ended and allow students to experience them at levels as high as they are capable.

Evaluation has shown that students traditionally labeled "slow" are doing better in CSMP classes than in non-CSMP classes. We believe this is due to the spiral development, to full group work on mathematical activities, and to the presence in the CSMP curriculum of certain nonnumerical activities in which many supposedly "slow" students can excel.

The slow learners should participate in math with the other students in the class, but they might also benefit from additional small group work. The additional time should not be simply to rehash or drill on the situations presented in class, but rather to provide additional experiences for these students. See the book "Math Play Therapy" Volumes I and II for suggestions of small group work with slow learners.

Additionally, CSMP provides for a variety of ability levels in the student materials. Worksheets and workbooks offer problems in a range from very easy to quite challenging. Indeed, some of the "higher starred" worksheets and pages toward the end of workbooks may be accessible only to your very best students.

In an extreme case, you might seriously consider the possibility of arranging for exceptionally bright students to take their math with a higher grade level CSMP class.

16. Q: IS IT NECESSARY TO FOLLOW THE SUGGESTED SCHEDULE EXACTLY?

A: This program is not meant to be a static sequence of exercises to be followed rigidly. The lessons in this guide are suggestions and the time allotments are merely approximations based on our own experience teaching the materials. If this will be the first time you are teaching the program and you would like specific suggestions for organizing the lessons, we offer some guidelines and a suggested schedule of lessons. You may wish to follow it at the beginning of the school year and then deviate from it as the needs of your students may dictate.

It is probably best to follow a one-day, one-strand pattern when you organize your own schedule, rather than to spend two or three full weeks "covering" one particular topic. Interchange the strands and the days of the week as you please. In a usual five day week, two days should be devoted to the World of Numbers strand, one day to the Workbook strand, and the other two days should be shared by the P, G, and L strands.

Exercise your own professional judgment in deciding when to stop a particular lesson. It is better to stop a lesson when student interest is high than to teach so methodically that everyone in the class understands everything, but is completely bored. In order to be successful with this curriculum, adjust the schedule to the ability, maturity, and interests of your class.

17. Q: HOW DO I HANDLE STUDENTS WHO HAVE AN EXTENDED ABSENCE OR STUDENTS (NEW TO CSMP) WHO TRANSFER INTO MY CLASS DURING THE YEAR?

A: The spiral approach allows a student to miss one or two weeks of school without much "catch up" work needed. When the student returns, either you or classmates can help with any material that is deemed necessary.

If several months of classes have been missed, you might need to treat the student as you would a tranfer student new to CSMP. Give the student a little extra attention during a lesson especially when the questions are review for most of the class.

In most cases, the differences between new and veteran students tend to fade rather quickly. One exception might be in lessons involving the Minicomputer. Veteran students who are unusually adept with the Minicomputer very often are good tutors for the transfer students in the use of this abacus. In addition, the storybook "Two By Two" is suggested as an aid for new students who are unfamiliar with the Minicomputer. Some of the storybooks and story-workbooks written for younger children can be used to introduce new students to the languages of strings and arrows.

It is probably best not to send worksheets and/or workbooks home to a child. Without participating in the collective lessons that introduce the worksheets and workbooks, it would be very difficult for the student to proceed.

18. Q: HOW DO I HANDLE WHOLE CLASSES WHICH ARE NEW TO CSMP?

A: CSMP does not assume that Kindergarten and first grade students have any prior experience with CSMP. The Kindergarten and First Grade Programs will need no adaptation for classes which are new to CSMP.

Second grade classes which are new to CSMP will need to be introduced to the CSMP languages. For this purpose, a suggested adaptation of certain second grade lessons is included in the CSMP Mathematics for the Upper Primary Grades (UPG) Part I teacher's guide.

Third grade classes which are new to CSMP will need to become fluent in the CSMP languages before beginning many of the lessons in UPG Part III. A special schedule has been prepared for these classes which consists of prerequisite lessons and workbooks from

UPG Part I and II followed by a modified UPG Part III and IV program.

Fourth grade classes new to CSMP spend one semester with CSMP Mathematics for the Intermediate Grades (IG), Entry Module I, before beginning the regular fourth grade program. Similarly, fifth grade classes new to CSMP spend approximately one semester with an Entry Module II before beginning IG Part II. Future development plans provide for the creation of Entry Module III for entry into the CSMP curriculum at the sixth grade.

19. Q: WHAT SHOULD I DO WITH MY CLASS IF THEY DID NOT FINISH THE PREVIOUS YEAR'S SCHEDULE?

A: No special measures are required. Each part of the CSMP curriculum begins with introductory lessons. In addition, the spiral approach permits individual participation at the student's own level of understanding, even if that level is not quite as high as it might have been if the previous year's schedule had been completed.

20. Q: HOW CAN I ASSESS STUDENT PROGRESS AND ASSIGN GRADES?

A: CSMP does not believe that it is either desirable or reasonable for teachers to attempt to evaluate student mastery of skills from day to day or even from week to week. On the other hand, teachers, parents, and even the children themselves benefit by some monitoring procedure that records involvement in activities and assesses progress through the program.

Most teachers have their own monitoring devices with which they are comfortable. It is recommended that such devices be continued.

21. Q: HOW DO CSMP STUDENTS DO ON STANDARDIZED TESTS?

A: While CSMP disagrees with the philosophy underlying standardized testing, test results are available to us from several sources. The results indicate that CSMP classes generally do as well or better on tests than do non-CSMP classes. It should be pointed out that standardized testing and traditional programs are correlated and that the CSMP student has been exposed to a great deal of mathematics that is not taught in more traditional programs or considered in the construction of standardized tests. There are also many unmeasureable or unmeasured areas such as the enjoyment of mathematics, strategic thinking, organizational ability, and depth of understanding of the methods and uses of mathematics where we feel CSMP students have a definite advantage. See also the comments on Evaluation in Section 2 (d) of the General Introduction.

22. Q: WHAT PROBLEMS WOULD CSMP STUDENTS HAVE IF THEY RETURN TO A MORE TRADITIONAL PROGRAM IN THE NEXT SCHOOL YEAR?

A: There should be no problems for CSMP students who return to a traditional program. Test results indicate that CSMP students are at or above the level of non-CSMP students in material covered in traditional courses. In addition, CSMP students have encountered many situations that are not introduced in traditional programs nor tested for on standardized tests. (See Question 21) In particular, their reasoning ability should be much sharper than their non-CSMP counterparts.

There are, of course, differences in timing as to when different programs introduce the standard algorithms and often CSMP's introduction will occur later than in some of the other programs. The reason for this is our belief that it is essential that the creative freedom of students not be impaired by a premature emphasis on the standard algorithms for adding, subtracting, multiplying, and dividing. We believe that many of the present computational difficulties experienced by elementary and junior high school students were caused by their spending an inordinate amount of time trying to master the algorithms too early in their mathematical education. This shuts the students off from other experiences necessary for their mathematical growth. Students need plenty of time to explore the world of numbers. Mastering an algorithm should be the last step of this exploration, a sort of synthesis of previous experience. Furthermore, as electronic hand-calculators become available to nearly everyone, algorithmic mastery as a prerequisite to other numerical experiences should become less important.

CSMP students can usually solve a problem before having learned the standard algorithm. If they enter a program that expects an algorithm they have not learned, they are still more than adequately prepared for the algorithm and should have no trouble quickly learning to use it.

CSMP introduces the usual addition algorithm in the second grade, one of the two standard subtraction algorithms in the third grade, the multiplication algorithm in the fourth grade, and the division algorithm in the fifth grade.

23. Q: WHAT ARE SOME WAYS TO ORGANIZE MATERIALS AND MANAGE MY CLASS?

A: It is very important to organize your CSMP materials before school begins. Decide where you will store the materials once they are organized. Filing drawers, shelves, boxes, or closets are all satisfactory as long as you can easily get to the materials. Take them all out of their shipping boxes and spread them on several tables and/or desk tops in your classroom so they can be organized into the order you will be using them.

Workbooks: The teacher's guide is needed to find the order in which the workbooks are used. Locate the Workbook strand column in the

Master Suggested Schedule of Lessons and collect all the workbooks in the order in which they are listed. After organizing the workbooks in order, place them in an upright position in your file drawer, box, shelf, etc., so that the first workbook to be used is at the front of the stack.

Worksheets. There is a separate worksheet booklet for each student and you should decide whether you will store these booklets and pass them out when a particular lesson calls for a worksheet, or simply let the students keep their own worksheet booklets in their desks.

Each worksheet can be removed from the booklet or left in depending on how you wish to manage worksheets. However, once a worksheet is removed from the booklet it cannot be replaced, so make sure students are careful about tearing off the correct sheet.

Manipulatives: Storage of each manipulative in a separate container, properly marked, is one way to make everything accessible. Another way is to provide a sturdy box (for example, a shoe box) for each student and to put individual manipulative materials in the box, properly labeled with the student's name.

Additional Materials: Be sure that you have plenty of high quality colored chalk, large sheets of unlined white paper, and that your students have colored pencils, pens, or crayons. Plan well ahead to be sure you have any needed materials that are not supplied by CSMP.

Keep all teacher materials where they are easily accessible. You may find the guides easier to use if they are put in looseleaf binders; the books are preferated so that pages can be removed easily. The demonstration Minicomputer boards and number line should always be on display in the room.

24. Q: SOME OF THE CSMP MATERIAL IS NEW TO ME. HOW DO I LEARN ENOUGH TO TEACH A CLASS?

A: Every CSMP teacher should have been trained by his or her coordinator. Kindergarten and first grade teachers receive a minimum of 18 hours training, teachers of the Upper Primary Grades Program receive up to 32 hours training, and teachers of the Intermediate Grades Program receive up to 56 hours training. During the training sessions the CSMP content is taught to new teachers and suggestions are given for organization of their classes.

If the suggested training schedule has been followed, many lessons taken from the CSMP curriculum, or very similar lessons, have been presented. The training schedule also makes use of the spiral approach so that training sessions aim not only to teach CSMP mathematics but to familiarize teachers with the spirit and pedagogy of the CSMP curriculum.

25. Q: WHAT SHOULD BE DONE WHEN THERE IS A SUBSTITUTE TEACHER?

A: Some school systems have included people who frequently substitute in their CSMP training program. When this is not the case, it might be very difficult for the untrained substitute to teach from CSMP materials. One solution is to arrange for another CSMP teacher to teach math to your class while the substitute teacher teaches another subject to that teacher's class. If that can not be arranged and you will only miss one or two days, you may find certain CSMP material, uncompleted workbooks for example, that the substitute can manage. Otherwise, use those days to cover material not included in the CSMP curriculum, such as telling time. If your absence will be an extended one, arrangements should be made with your coordinator to train a substitute, rearrange schedules to put a CSMP teacher in your class, or otherwise provide for your students.

26. Q: THE PROGRAM MAKES USE OF THE STUDENT'S CREATIVE FREEDOM. HOW MUCH ROOM IS THERE FOR THE TEACHER'S INGENUITY?

A: CSMP not only allows for your ingenuity but depends on it. While the lessons may appear to be very rigid by presenting dialogue for teacher and student, it should be remembered that these are suggested questions, answers, and comments based on our experiences with development classes. Even if you were to follow your part of the "script", your students are likely to offer responses which differ from those we have listed. It is the student's response that will determine the content and direction of your next question or comment.

In a traditional program your creativity is called upon the first time you develop a particular lesson, and to a lesser extent, as you revise lessons in succeeding years. There is little room for spur-of-the-moment creativity once a lesson begins. The CSMP curriculum, by creating a free and open situation emphasizing the student's creative freedom, continually presents you with unanticipated student responses and calls on your ingenuity at all times.

27. Q: HOW CAN I EXPLAIN CSMP TO PARENTS?

A: The best way would be for your coordinator to organize a workshop for interested parents. The parents can then see the languages of the program in use and have some of the same experiences their children have. Workshops can be of almost any length but anything short of two hours does not allow for much familiarity with the material. In such a workshop, we suggest that the language of the Minicomputer be explained with particular emphasis on its use in presenting situations where mental arithmetic is called upon and in developing algorithms (for example, physically carrying a checker from the ones' to the tens' board in the "8 + 2 = 10" play, paralleling "carrying" in the paper and pencil algorithm). A lesson using the storybook "Where's My Nose?" provides a good example of the power of the language of strings. A nonnumerical example, such as the brother-sister graph, and a numerical example, such as building a road from 5 to 55 using +3 and 5× arrows, will provide an introduction to the language of arrows. The lessons that are the favorites of the children would be excellent for a presentation to parents as well.

You might also extend an invitation to parents to visit your class during the math period. Many times such an invitation leads eventually to a closer parent-teacher relationship, with the parent offering voluntary services as an aide.

You will find a letter to parents at the end of this section which may be reproduced and sent home with the students.

TO THE PARENTS OF CSMP STUDENTS

Your child is participating in the Comprehensive School Mathematics Program's (CSMP) elementary school curriculum, developed under a contract with the National Institute of Education. While the program cannot be fully explained in a brief note, we hope that this letter will give you some feeling for the substance of your child's mathematics education.

The content goals of the CSMP K-6 curriculum include a development of the standard number systems and their operations (including an early introduction to the integers, rational numbers, and decimal numbers) interwoven with a development of geometry and measurement. On an equal footing with the above, the CSMP curriculum includes a strong component of probability, statistics, and combinatorics which extends throughout grades 2-6. Powerful pedagogical devices are used, such as several nonverbal languages. These make the unifying ideas of the program accessible to very young children.

These languages include:

1) The Language of Strings This "language" mimics the basic methods of collecting and classifying data related to the problems at hand. That is, it deals with the fundamentally useful and important mathematical notion of set.

2) The Language of Arrows This graphical language models the process of comparing and analyzing sets and operations on them. That is, it deals with the mathematical notions of relations and functions including, but not restricted to, ordering relations and the numerical processes of adding to, subtracting from, multiplying by, and sharing equally among.

3) The Language of the Papy Minicomputer The Papy Minicomputer, an extremely effective paper abacus invented by Georges Papy, models the positional structure of our system of numeration and lends itself to a multitude of numerical algorithms, including the standard algorithms for the basic numerical operations.

Among other things, these languages allow for the development of the essential operations on numbers in a natural way based on previous experiences with the languages. They also provide a context in which situations arise that call for the repeated use of arithmetic operations without resorting to drill exercises, which most children find boring.

Another tool, not unique to but used by CSMP, is the hand-calculator. The hand-calculator allows the child to study problems that call for calculation beyond the child's ability or in which the calculation is so extensive as to cause the child to lose interest. The calculator is also used in a way similar to the use of languages mentioned above. Properly used, it can help lead to the development of important mathematical ideas and present situations for the interesting use of mental or paper and pencil arithmetic.

The content is learned in an atmosphere of constant applications utilizing a "pedagogy of situations".* These vary from simply story situations in the early primary grades to challenging applications and nontrival simulations of real world problems in the upper grades.

One of the tenets of CSMP's development philosophy is that mathematics is a unified whole and should be learned as such. As a result, along with CSMP's concern with the variation in ability levels in the typical heterogeneous class, the curriculum is structured in a spiral format throughout the K-6 program. The content is completely sequenced in spiral form so that each student is brought into contact with each area of content continuously throughout the program. This consciously avoids atomizing the content and mastering each bit before continuing to the next. Rather, each student spirals through repeated exposures to the content, building interlocking experiences of increasing sophistication.

Another tenet of CSMP's development philosophy, based on years of experimentation in the classroom, is that no single method of classroom management suffices to meet the needs of every student. Hence, the program is constructed to allow numerous opportunities for whole class participation, small group participation, and individual independent experiences. Individualization throughout the K-6 program is achieved through the independent work on workbooks and worksheets of varying levels of difficulty. Due to the spiral nature of the curriculum, each student can engage in a workbook without losing touch with the progress of the class while at the same time working to the limits of his or her ability.

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* Inspired by the work of the mathematician and psychologist, Caleb Gattegno.

A feature unique to the CSMP program is a collection of storybooks written by Frederique Papy (STORIES BY FREDERIQUE) and a collection of story-workbooks (THE CSMP LIBRARY). These probe the affective domain of learning and bring children into contact with mathematics at levels until now unexplored.

CSMP does not have its own system of student assessment and progress reporting. While standardized testing indicates that CSMP students develop the "basic skills" as early or earlier than do non-CSMP students, we hope you will not be obsessed with your child's development of these "skills". Each child is an individual and should not be judged against norms but on what he or she can and does accomplish. Many extremely important areas -- development of self-confidence, ability to think and reason, etc. -- are very difficult to measure and grade. If you have any questions regarding the progress of your child, his or her teacher is in the best position to answer those questions.

THE CSMP STAFF

WORKBOOKS

Comprehensive School Mathematics Program

Mathematics for the Intermediate Grades
Part IV

Workbooks

CEMREL, Inc.

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WORKBOOK

INTRODUCTION

There are many opportunities for the students to work individually during the course of the lessons described in the four content strands. In the workbook strand, however, it is this individualized work which becomes the chief end of most of the lessons. It has been our goal in this strand to provide the students with opportunities to

- review many of the ideas they have met in the content strand;
- apply their acquired knowledge to new situations requiring various kinds of strategic thinking;
- learn how to read and use mathematics workbooks and story-workbooks.

The W - strand includes the following workbooks.

A Selection of Problems #1
A Selection of Problems #2
A Selection of Problems #3
A Selection of Problems #4
A Selection of Problems #5
A Selection of Problems #6

Each of these six workbooks has problems of varying levels of difficulty all in one booklet. Approximately the first ten pages of each booklet are easy problems, the next ten to twelve pages are average problems, and the last ten pages are challenging problems.

The W - strand also includes the following storybooks.

A Valentine Mystery
The Hidden Treasure

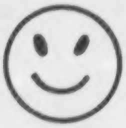
We suggest that you devote one day a week to this strand as indicated on the suggested schedule of lessons in the General Introduction. We also recommend that you follow this schedule more or less as given, even if many of your students have not finished a particular workbook after the two periods that are alotted to it. As a general rule, students at this age do not enjoy remaining on the same topic for too long. It is a basic principle of the CSMP spiral approach that a variety of experiences is preferable to any attempt to master a given topic by continued "plugging away" at it. This principle applies as much (if not more so) to the Workbook strand as it does elsewhere in the curriculum. If you do feel, however, that more time is needed for a particular workbook, you could allow students to continue their work on that workbook during one (or more) of the adjustment days or during their free time.

The placement of the various workbooks in the suggested schedule of lessons was made keeping in mind where lessons from the other content strands occur. The types of problems in each workbook reflect the topics covered in earlier lessons and assume information has previously been introduced. Hence, you are warned to be particularly careful when considering possible modification of this suggested schedule. Whatever changes you might like to make, we strongly urge that you use one day each week for workbooks.

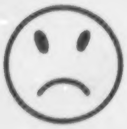
For each of the storybooks in this Workbook strand, there are collective lessons that accompany the storybook. There are also short collective (warm-up) lessons suggested for some of the "A Selection of Problems" workbooks. Following any such suggestions for a particular workbook, there is an answer key for the workbook. All answer keys are presented in reduced format, each page actually containing reduced copies of four of the pages of the original booklets, with solutions written in.

For each workbook we suggest that all students start work at the easiest level of difficulty and then work through as many pages as they can handle during the two lessons scheduled to that workbook. You can expect that, under normal circumstances, about two-thirds of the students will work through about 20 to 25 pages, and about one-third will reach the most challenging problems at the end of the booklet.

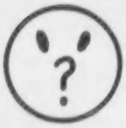
The workbooks provide an excellent instrument with which you can assess the progress of your students on a week-by-week basis. To use this instrument effectively certainly does not require that you check ever page and every problem for each student. You should develop the sort of procedure for checking students' work with which you are most comfortable. For example, you might check one or more specific pages, discuss some particular mistakes with individual students, and let them correct their work before continuing. Or you might simply look carefully at the first few pages of the workbooks to see that the students understand the general idea of the problems. In our experience, an effective procedure is to clip to the front cover of each checked workbook a strip of paper on which can be indicated your assessment of the students' work on each page you check. For this purpose, you might use the following symbols.



meaning "well done" or "correct"

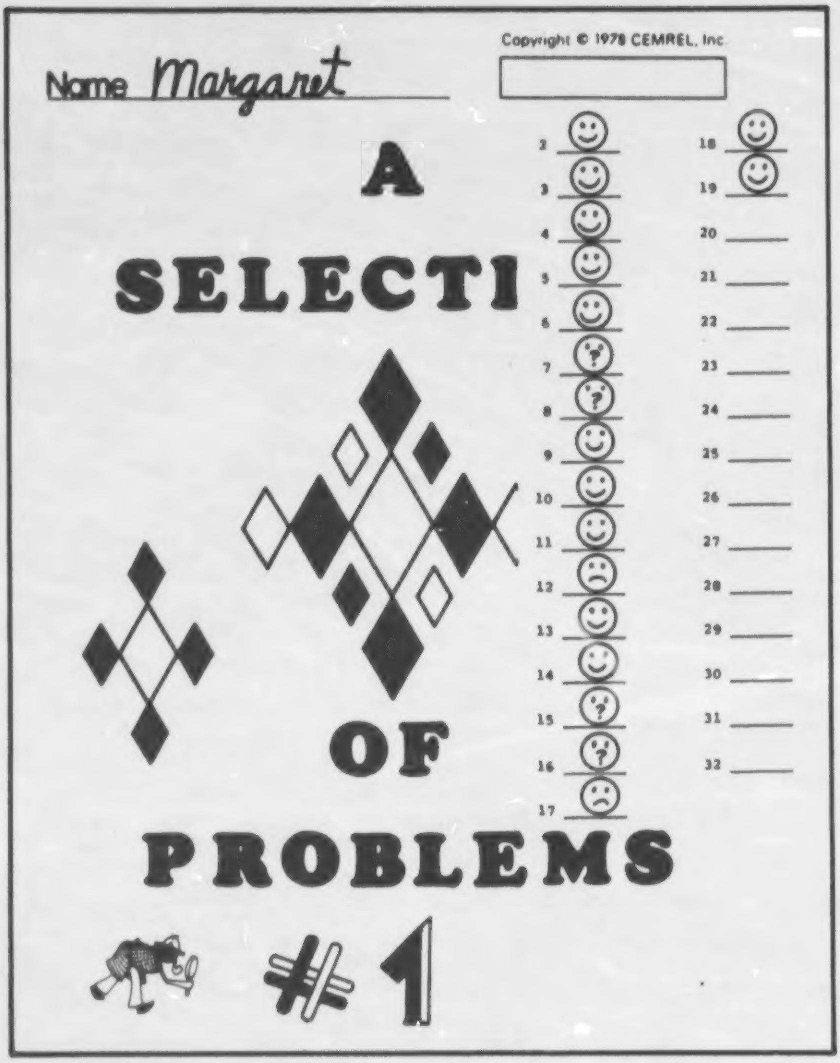


meaning "something wrong, try again"



meaning "did you forget this problem?"
or "do you need a hint?"

On the next page we show the front cover of one student's A Selection of Problems #1 workbook after it has been checked.



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Name *Margaret*

A

SELECTI

OF

PROBLEMS

#1

2 ☺
3 ☺
4 ☺
5 ☺
6 ☺
7 ?
8 ?
9 ☺
10 ☺
11 ☺
12 ☹
13 ☺
14 ☺
15 ?
16 ?
17 ☹
18 ☺
19 ☺
20 ____
21 ____
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31 ____
32 ____

NOTE: This is the way Margaret's workbook was returned to her at the beginning of the second lesson devoted to this workbook. The blanks on lines 20 - 32 indicate that she had not yet worked on these pages. Before Margaret begins on these pages during the second lesson, she should correct her mistakes on pages 12 and 17 and finish her work on pages 7, 8, 15 and 16.

Immediately following this introduction to the W - strand, we provide a progress chart to help you keep track of

- where each of your students is on any particular workbook day;
- how many pages each student has completed under each title;
- your personal degree of satisfaction with the student's work on each workbook.

In particular you should be on the watch to see if, for any of your students, even the easiest problems in the workbooks are too difficult. Such students would probably benefit a great deal if you were able to devote some time with them on the activities suggested in the book "Math Play Therapy".

Here is a list of some of the important points to bear in mind when working in this strand.

- All students start at the beginning of each workbook and progress as far as they can.
- All students begin a new title on the same day, even if some students have not finished work on the previous title.
- Not all students are expected to complete a given workbook. Only the faster stüdents will reach the most challenging problems. The slower students may succeed to do only the easiest problems although you should not assume that this will automatically be the case—surprises are not at all uncommon in this area.
- Read the introductory material for each workbook and storybook and give the full group lesson (s) when appropriate.

Workbook Progress Chart

Name	A Selection of Problems #1	A Selection of Problems #2	A Selection of Problems #3

Workbook Progress Chart

Name	A Selection of Problems #4	A Selection of Problems #5	A Selection of Problems #6

A SELECTION OF PROBLEMS

#1

A SELECTION OF PROBLEMS #1

The problems in this workbook are in order of increasing difficulty. Approximately the first ten pages of the workbook have easy problems, the next ten to twelve pages have average problems, and the last ten pages have more challenging problems. It is suggested that all students work through as many pages as they can handle during the two lessons scheduled for the workbook. Not all students are expected to complete the entire workbook; some will succeed to do only the easy problems, many should finish about twenty pages, and only the faster students will reach the most challenging problems at the end of the booklet.

Many problems in the workbook were designed with the availability of hand-calculators in mind. If possible, provide all the students with hand-calculators or at least give students access to hand-calculators.[*] You will find that many students do not need or want to use calculators; however, some students still need the support of the calculator in thinking about these problems.

A brief collective activity is suggested to begin each of the workbook lessons. These should be short warm-up activities which serve to review terms or to present a situation similar to one in the workbook that might cause some difficulty.

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[*] See Section 2(f) of the General Introduction for additional comments on the use of hand-calculators and on desirable characteristics of hand-calculators.

W 1

Lesson 1

Write "Square Numbers" on the board.

T: What are some square numbers?

S: 25, because $5 \times 5 = 25$.

S: 4, because $2 \times 2 = 4$.

Invite the students to name many square numbers and list their suggestions on the board.

T: If a secret number, Flip, is a square number, what do we know about Flip?

S: We know that some whole number times itself equals Flip.

T: Who can name some large square numbers?

S: 900, because $30 \times 30 = 900$.

S: 10,000, because $100 \times 100 = 10{,}000$.

T: What is the smallest square number?

S: 0, since $0 \times 0 = 0$.

Erase the board and write "Positive Divisors".

T: What are the positive divisors of 12?

S: 6, because $2 \times 6 = 12$. It also means that 2 is a positive divisor of 12.

S: 12 and 1 since 12 × 1 = 12.

S: 3, because if you count by three's from zero, you will hit 12.

S: 4, because 4 × 3 = 12.

Ask the students for the positive divisors of a few more numbers, for example, 18, 25, and 28.

T: What are the positive divisors of 11?

S: 11 and 1, since 11 × 1 = 11. These are the only positive divisors of 11.

T: What do we call a number like 11 that has exactly two positive divisors?

S: A prime number.

T: What are some other positive prime numbers? (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and so on.)

Is 1 a prime number?

S: No, because 1 has only one divisor, namely itself.

Give each student a copy of the workbook "A Selection of Problems #1" and let them work on their own.

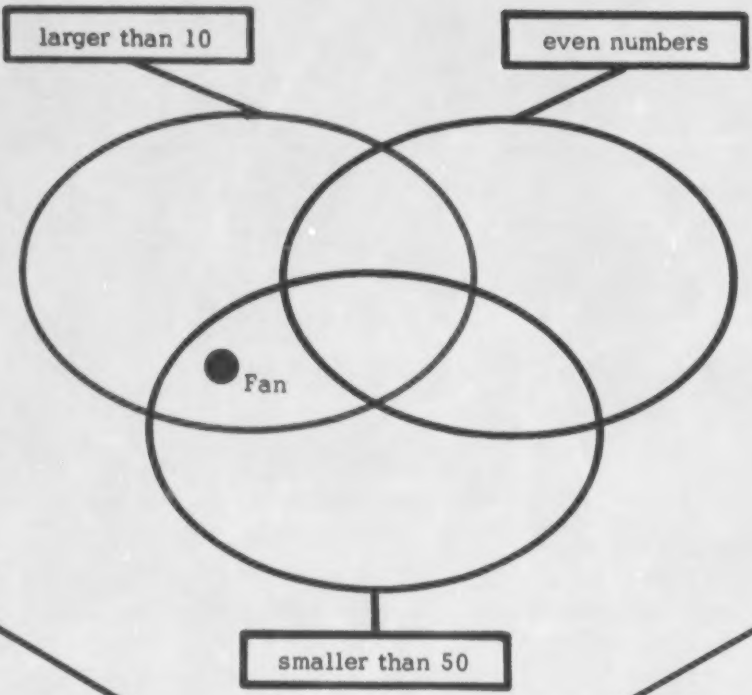
If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

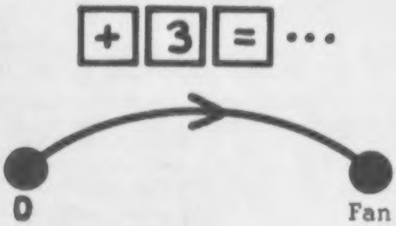
At the end of the lesson, collect the workbooks and have them ready to return for Lesson 2.

W 2

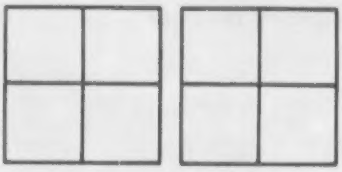
Lesson 2

Write all three of these clues on the board.


larger than 10
even numbers
Fan
smaller than 50



Fan can be put on this Minicomputer with exactly two regular positive checkers.



T: Fan is a secret number. What can you tell me about Fan?

S: Which clue should we use first?

T: You may use the clues in any order you wish.

Let the students determine how to proceed to find Fan. If the class has difficulty with one clue, encourage them to try a different clue first. There are several approaches your class might take. The following dialogue indicates one way the discussion might proceed.

S: The string picture shows that Fan is between 10 and 50.

S: Also, Fan is outside the red string, so Fan is an odd number between 10 and 50.

S: Fan could be 11, 13, 15, 17, and so on up to 49.

Write this list of numbers on the board.

11, 13, 15, 17, 19, . . . , 47, 49

S: Fan can be put on the Minicomputer with exactly two regular positive checkers, so Fan could be 11, but not 13, 15, or 17.

S: Fan could only be 11, 21, or 41. These are the only numbers in the list from the string picture clue that also satisfy the Minicomputer clue.

Write these numbers on the board.

11, 21, 41

S: Fan is 21 because the arrow clue tells us that Fan is a multiple of 3. 21 is a multiple of 3, but 11 and 41 are not multiples of 3.

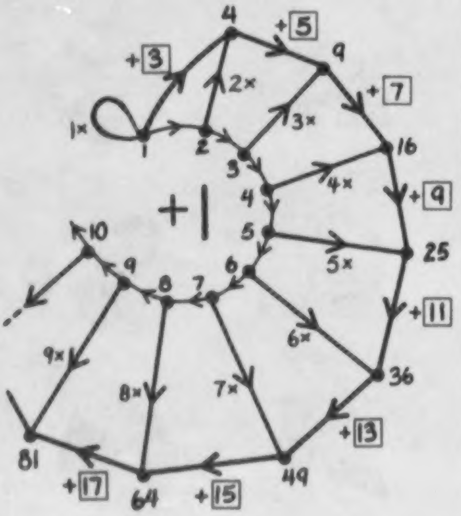
Return the students' copies of "A Selection of Problems #1". Ask the students to correct their mistakes, if any, and then to continue working in their workbooks.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion of the problem.

At the conclusion of this lesson, collect the workbooks for your progress check. You may wish to let some students work further in their workbooks later during any free study time.

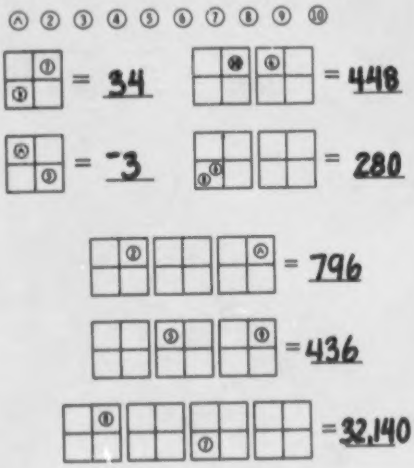
A SELECTION OF PROBLEMS #1

Label the dots and fill in the box for each green arrow.



+1
1×
1
2
3
4
5
6
7
8
9
10
2×
3×
4×
5×
6×
7×
8×
9×
4
9
16
25
36
49
64
81
+3
+5
+7
+9
+11
+13
+15
+17

Put any number you wish on the Minicomputer using exactly two of these checkers.

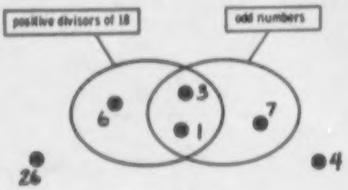


^ ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩
= 34
= 448
= ^-3
= 280
= 796
= 436
= 32,140

(Many solutions are possible.)

Put each of these numbers in the string picture.

3 4 6 7



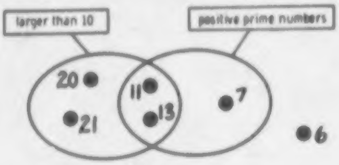
positive divisors of 18
odd numbers
6
3
1
7
26
4

Label the other two dots with numbers of your choice.

(Many solutions are possible.)

Put each of these numbers in the string picture.

6 7 13 20

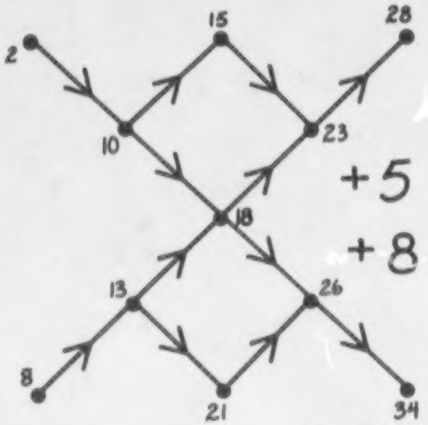


larger than 10
positive prime numbers
20
21
11
13
7
6

Label the other two dots with numbers of your choice.

(Many solutions are possible.)

Label the dots.



2
15
28
10
23
+5
+8
18
13
26
8
21
34

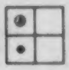
Which is the smallest odd number in this picture? 13

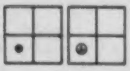
Which is the largest multiple of 4 in this picture? 28

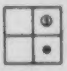
Lam is a secret number.

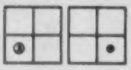
First clue

Lam is one of these numbers.

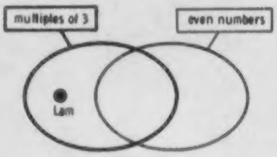

= 18


= 32


= 21

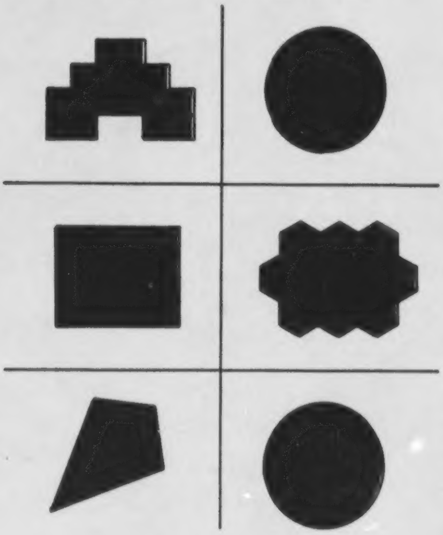

= 61

Second clue


multiples of 3
even numbers
Lam

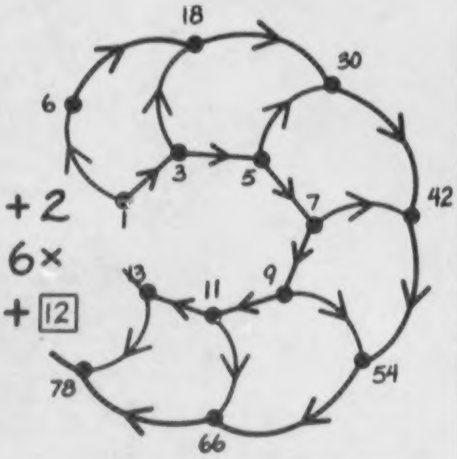
Who is Lam? 21

Color one-half of each shape red and one-half of each shape blue.



(Many solutions are possible.)

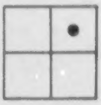
Label the dots and fill in the box for the green arrows.


18
30
6
3
5
7
42
1
+2
6×
+ 12
9
13
11
54
78
66

Clop is a secret number.

First clue

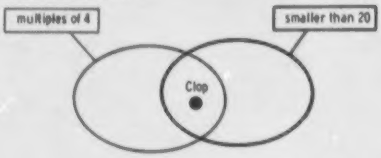
Clop can be put on this ones' board of the Minicomputer by adding exactly one ③-checker.



Clop could be 7 or 10 or 16 or 28.

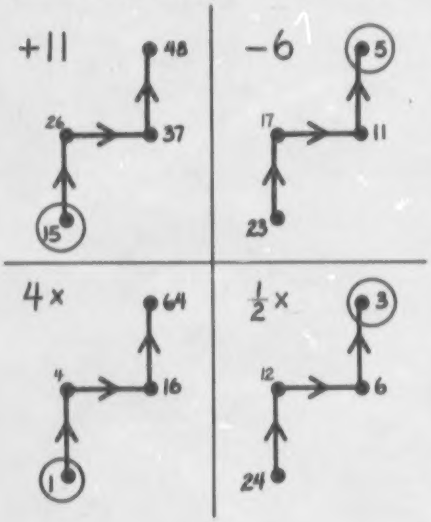
Second clue

Clop is in this string picture.


multiples of 4
smaller than 20
Clop

Clop is 16.

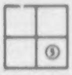
Label the dots.
Exactly one number in each picture is a positive divisor of 15. Circle its dot.


+11
48
26
37
15
−6
5
17
11
23
4×
64
4
16
1
$\frac{1}{2}$×
3
12
6
24

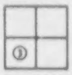
Put each number on the ones' board of the Minicomputer using exactly one of these checkers. Many answers are possible.

② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

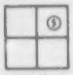
An odd number


= 5
(3, 7, and 9 are also solutions.)

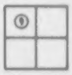
A positive divisor of 30


= 6
(2, 3, 5, and 10 are also solutions.)

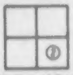
A multiple of 10


= 20
(10 and 40 are also solutions.)

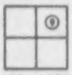
A number larger than 50


= 72
(56 and 64 are also solutions.)

A positive prime number


= 2
(3, 5, and 7 are also solutions.)

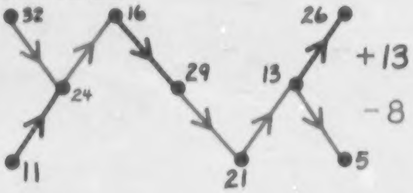
A square number


= 36
(4, 9, 16, and 64 are also solutions.)

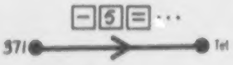
Tet is a secret number.

First clue

Tet is in this arrow picture.


32
16
26
+13
24
29
13
−8
11
21
5

Second clue



Tet could be 11 or 16 or 21 or 26.

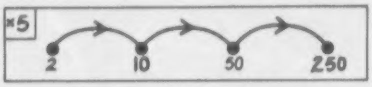
Third clue

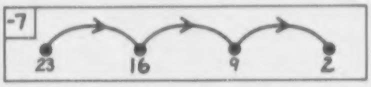
Tet is a positive prime number.

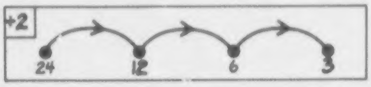
Tet is 11.

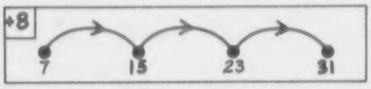
For each arrow picture, fill in the box with the correct tag. Then label the dots.

$\boxed{+8}$ $\boxed{-7}$ $\boxed{\times 5}$ $\boxed{\div 2}$

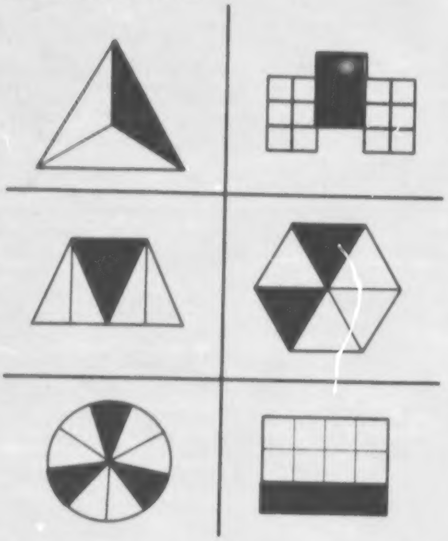








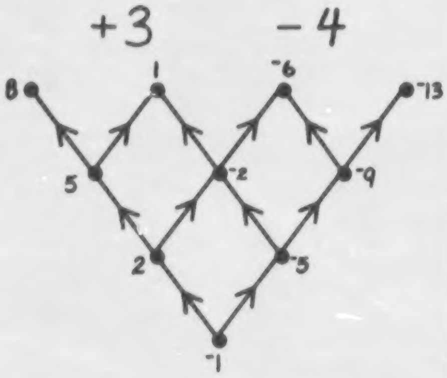
Color one-third of each shape red.



14

(Many solutions are possible.)

Label the dots.


+3
−4
8
1
⁻6
⁻13
5
⁻2
⁻9
2
⁻5
⁻1

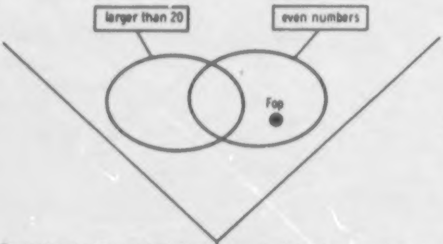
Which is the largest number in this arrow picture? 8

Which is the smallest number in this arrow picture? ⁻13

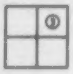
Which is the largest negative number in this arrow picture? ⁻1

15

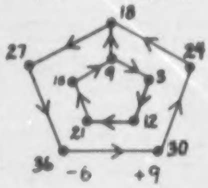
Fop is in this string picture.


larger than 20
even numbers
Fop

Fop can be put on the ones' board of the Minicomputer using exactly one ③-checker.



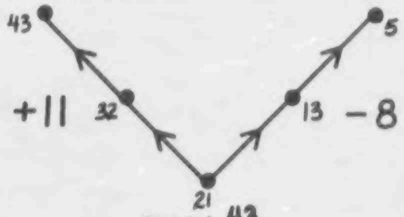
Fop is in this arrow picture.


18
27
24
18
9
3
21
12
36
30
−6
+9

Who is Fop? 12

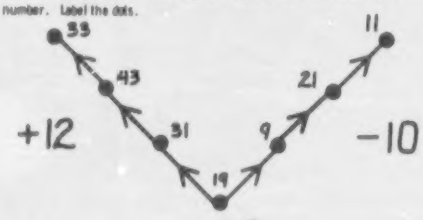
16

In this arrow picture, 5 is the smallest number and Pak is the largest number. Label the dots.


43
5
+11
32
13
−8
21

Who is Pak? 43

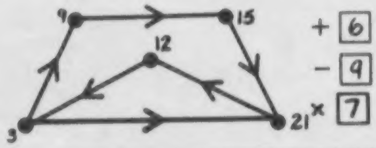
In this arrow picture, 43 is the largest number and Tak is the smallest number. Label the dots.

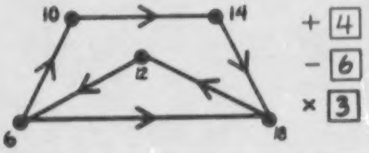

55
11
43
21
+12
31
9
−10
19

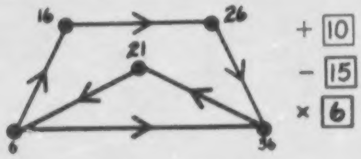
Who is Tak? 9

17

Label the dots and fill in the boxes for the arrows.

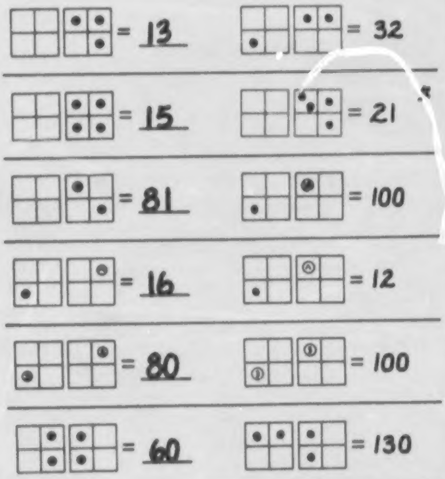

9
15
12
3
21
+ 6
− 9
× 7


10
14
12
6
18
+ 4
− 6
× 3


16
26
21
6
36
+ 10
− 15
× 6

18

Fill in the blanks on the left.
By moving exactly one checker, put the new number on the Minicomputer to the right.


= 13
= 32
= 15
= 21 *
= 81
= 100
= 16
= 12
= 80
= 100
= 60
= 130

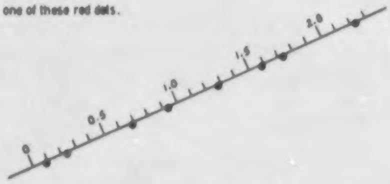
19

* (Moving the checker from the 4-square to the 10-square is another solution.)

Nu is a secret number.

First clue

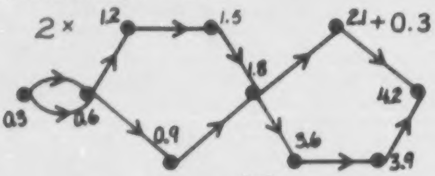
Nu is one of these red dots.



Nu could be 0.05, 0.2, 0.65, 0.9, 1.25, 1.55, 1.7, or 2.2.

Second clue

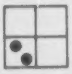
Nu is in this arrow picture. Label the dots.

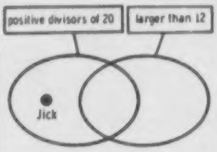

2×
1.2
1.5
2.1
+0.3
0.3
0.6
0.9
1.8
4.2
3.6
3.9

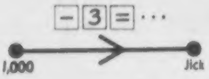
Who is Nu? 0.9

20

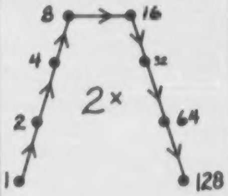
Jick can be put on the ones' board of the Minicomputer with exactly two regular positive checkers.




positive divisors of 20
larger than 12
Jick



Jick is in this arrow picture.


8
16
4
32
2×
2
64
1
128

Who is Jick? 4

21

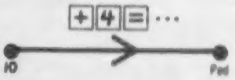
Ped is a secret number.

First clue

Ped is a four digit number with exactly two 4's and two 6's. For example, Ped could be 4,664.

Ped could be 4,466, 4,646, 4,664, 6,446, 6,464, or 6,644.

Second clue



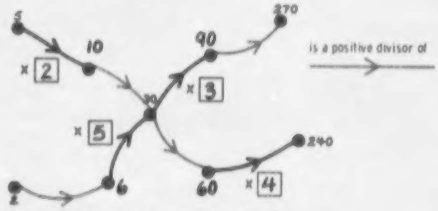
Ped could be 4,466 or 4,646 or 6,446.

Third clue

Ped is larger than 79 x 79.

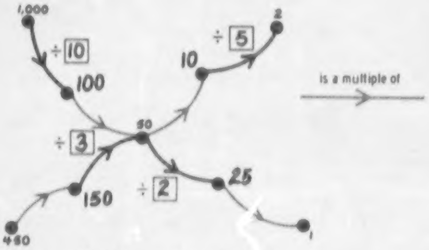
Ped is 6,446.

Label the dots. For each red arrow, fill in the box with a WHOLE number. Many answers are possible.



5
10
× 2
90
270
30
× 3
× 5
240
2
6
60
× 4
is a positive divisor of

(Many solutions are possible.)



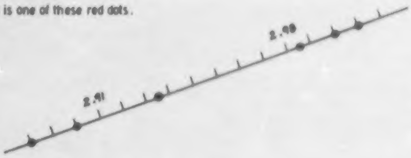
1,000
÷ 10
100
10
÷ 5
2
is a multiple of
50
÷ 3
150
÷ 2
25
450
1

(Many solutions are possible.)

Bo is a secret number.

First clue

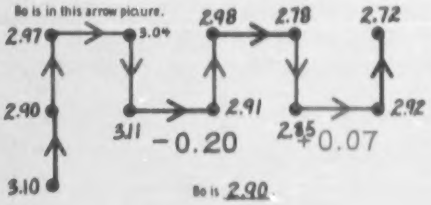
Bo is one of these red dots.



Bo could be 2.88, 2.90, 2.935, 2.985, 3.00, or 3.01.

Second clue

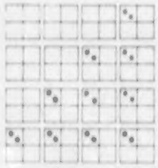
Bo is in this arrow picture.



2.97
3.04
2.98
2.78
2.72
2.90
3.11
2.91
2.85
2.92
3.10
−0.20
+0.07

Bo is 2.90.

Using only regular positive checkers, put these numbers on the Minicomputer.

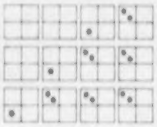


Use exactly two checkers: = 16

Use exactly four checkers: = 176

Use exactly six checkers: = 1,776

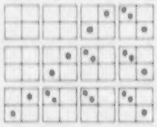
Use exactly eight checkers: = 17,776



Use exactly three checkers: = 36

Use exactly five checkers: = 376

Use exactly seven checkers: = 3,776



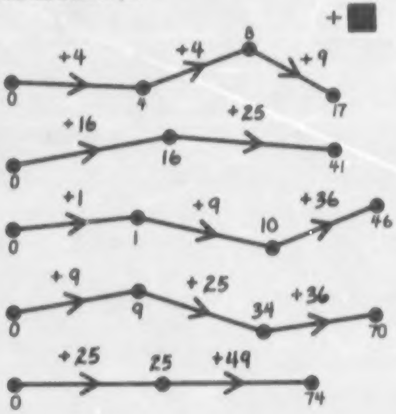
Use exactly five checkers: = 77

Use exactly seven checkers: = 777

Use exactly nine checkers: = 7,777

What are the ten smallest SQUARE numbers? 0 , 1 , 4 , 9 , 16 , 25 , 36 , 49 , 64 , 81 .

" + ■ " means each red arrow is for plus some SQUARE number.
In these six arrow roads, label the dots and label each red arrow " + ☐ ".
The first road is done for you.



(The above solutions have the arrows in increasing order. Any rearrangement of the arrows will yield other correct solutions, provided the dots are labeled accordingly.)

$\boxed{3}$ $\boxed{1}$ $\boxed{4}$ $\boxed{2}$

What is the smallest four digit number with these four digits? 1,234

What is the largest four digit number with these four digits? 4,321

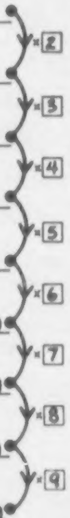
List all the four digit numbers with these four digits. (There may be extra blanks.)

1,234 . 1,243 . 1,324 . 1,342 . 1,423 . 1,432 . 2,134 .
2,143 . 2,314 . 2,341 . 2,413 . 2,431 . 3,124 . 3,142 .
3,214 . 3,241 . 3,412 . 3,421 . 4,123 . 4,132 . 4,213 .
4,231 . 4,312 . 4,321 . ____ . ____ . ____ . ____

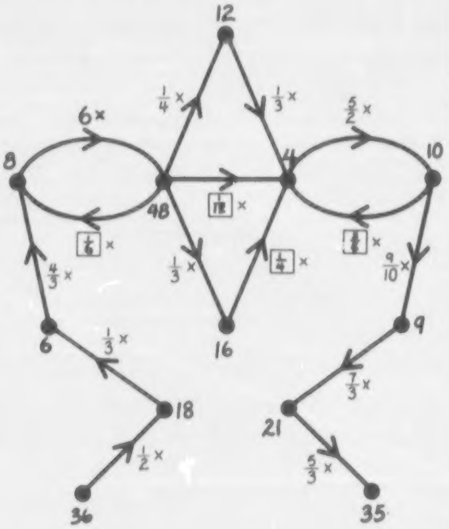
How many four digit numbers did you list? 24

Fill in the blanks and the box for each red arrow.

How many one digit numbers are there using this digit exactly once? $\boxed{9}$ 1
How many two digit numbers are there using each of these digits exactly once? $\boxed{8}\boxed{9}$ 2
How many three digit numbers are there using each of these digits exactly once? $\boxed{7}\boxed{8}\boxed{9}$ 6
How many four digit numbers are there using each of these digits exactly once? $\boxed{6}\boxed{7}\boxed{8}\boxed{9}$ 24
How many five digit numbers are there using each of these digits exactly once? $\boxed{5}\boxed{6}\boxed{7}\boxed{8}\boxed{9}$ 120
How many six digit numbers are there using each of these digits exactly once? $\boxed{4}\boxed{5}\boxed{6}\boxed{7}\boxed{8}\boxed{9}$ 720
How many seven digit numbers are there using each of these digits exactly once? $\boxed{3}\boxed{4}\boxed{5}\boxed{6}\boxed{7}\boxed{8}\boxed{9}$ 5,040
How many eight digit numbers are there using each of these digits exactly once? $\boxed{2}\boxed{3}\boxed{4}\boxed{5}\boxed{6}\boxed{7}\boxed{8}\boxed{9}$ 40,320
How many nine digit numbers are there using each of these digits exactly once? $\boxed{1}\boxed{2}\boxed{3}\boxed{4}\boxed{5}\boxed{6}\boxed{7}\boxed{8}\boxed{9}$ 362,880



Label the dots and fill in the box for each red arrow.

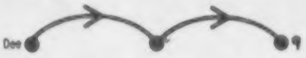


Dee is a secret number.

First clue

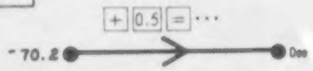
The BLUE arrow could be for:
10 ×
÷2
−3

The RED arrow could be for:
10 ×
÷2
−3



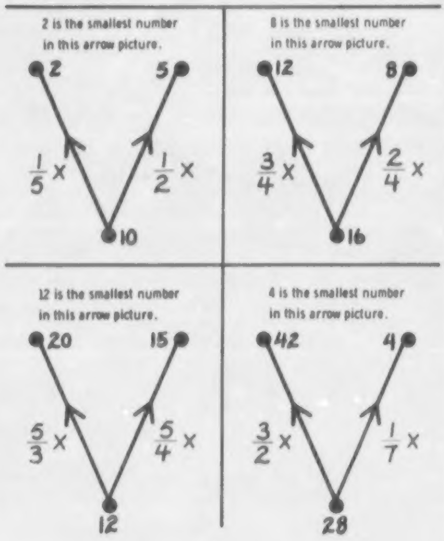
Dee could be 0.09, 1.2, 1.8, 3.9, 15, 21, 24, or 36.

Second clue



Dee is 1.8.

Label the dots.


2
5
1/5 x
1/2 x
10
12
8
3/4 x
2/4 x
16
20
15
5/3 x
5/4 x
12
42
4
3/2 x
1/7 x
28

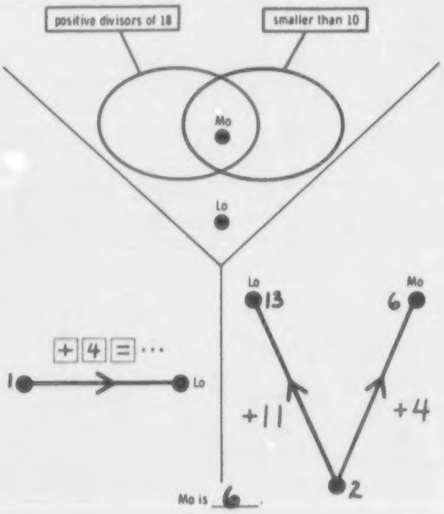
2 is the smallest number in this arrow picture.

8 is the smallest number in this arrow picture.

12 is the smallest number in this arrow picture.

4 is the smallest number in this arrow picture.

Mo and Lo are WHOLE numbers.


positive divisors of 18
smaller than 10
Mo
Lo
+ 4 = ···
1
Lo
Lo
13
Mo
6
+11
+4
2

Mo is 6.

Lo is 13.

A
VALENTINE
MYSTERY

A VALENTINE MYSTERY

Two lessons are based on the storybook, "A Valentine Mystery". In each lesson, the class reads the story collectively and stops to solve problems suggested in the storybook. The lesson descriptions indicate some questions and student responses that were made during the presentation of the storybook in our development classes. Feel free to modify the presentation in any way appropriate for your class.

Lesson 1

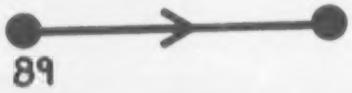
Distribute paper, colored pencils, and copies of the storybook, "A Valentine Mystery".

Pages 1 - 4

Read pages 1-4 together and encourage the class to discuss what kind of game that Zero might have invented for the numbers to play. Restate the rule of the game written in the red box on page 3.

Page 5

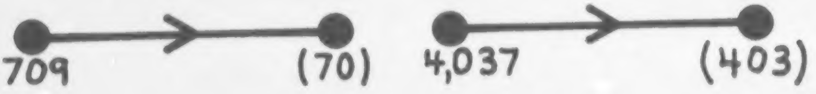
Read page 5 and then stop to pose some problems. For example, draw a red arrow starting at 89.



T: To whom does 89 send a valentine?

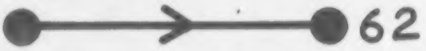
S: 8.

Similarly, ask to whom 709 and 4,037 send valentines.



W 3

Draw a red arrow ending at 62 on the board.



T: From whom could 62 receive a valentine?

S: 623.

T: Does 62 receive more than one valentine?

S: Yes, 62 receives valentines from 620, 621, 622, 623, 624, 625, 626, 627, 628, and 629. 62 receives exactly ten valentines.

Page 6

Ask the class to read this page on their own and to think about the questions.

T: Which numbers send valentines to 0?

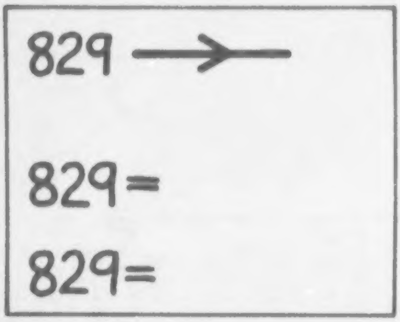
S: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

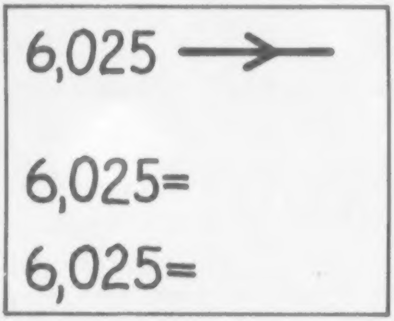
Pages 7 and 8

Ask your class to read these pages and to study the posters.

T: Use some of the extra space to draw your own poster for 829. When you finish the poster for 829, do one also for 6,025.

As the class does this, put this much of the posters on the board.





After most students have drawn their posters, invite students to complete the posters on the board.

$$\boxed{\begin{array}{l}829 \longrightarrow 82\\[1em] 829 = 820 + 9\\ 829 = (82\times 10)+9\end{array}} \qquad \boxed{\begin{array}{l}6{,}025 \longrightarrow 602\\[1em] 6{,}025 = 6{,}020 + 5\\ 6{,}025 = (602\times 10)+5\end{array}}$$

Write this problem on the board.

$$10\overline{)829}$$

T: Who can solve this division problem?

S: 82 with a remainder of 9.

Write the solution on the board with the label "quotient".

$$\begin{array}{l} \quad 82 \longleftarrow \text{R}=9 \ \ \text{quotient}\\ 10\overline{)829}\end{array}$$

T: We call this number (point to "82") in the division calculation the "quotient". In this calculation, the quotient is 82 and the remainder is 9.

Pages 9 and 10

Ask the class to read these pages as you write two problems on the board.

$$10\overline{)2{,}386} \qquad\qquad 10\overline{)7}$$

W 3

T: Who can solve these two division problems?

S: For the first problem, the quotient is 238 and the remainder is 6.

S: For the second problem, the quotient is 0 and the remainder is 7.

Write the solutions on the board.

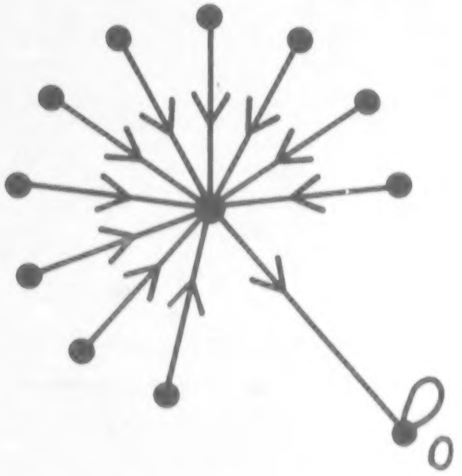
$$\begin{array}{r} 238 \\ 10\overline{)2,386} \end{array} \; R=6 \qquad\qquad \begin{array}{r} 0 \\ 10\overline{)7} \end{array} \; R=7$$

Pages 11 and 12

Ask a student to read these pages aloud. Note that the picture on page 12 is the completion of the poster on page 11.

T: On your paper, draw a flower like the one on page 12. Then make 0 the last dot at the end of the largest arrow and label the other dots appropriately.

As students work independently, draw this picture on the board.



T: What number could the center dot be for?

S: 6.

T: Then from whom does 6 receive valentines?

S: 60, 61, 62, . . . , 69.

T: Does everyone have the same solution?

S: No, I made the center dot 8. The other dots are for 80 to 89.

T: How many different flowers are possible?

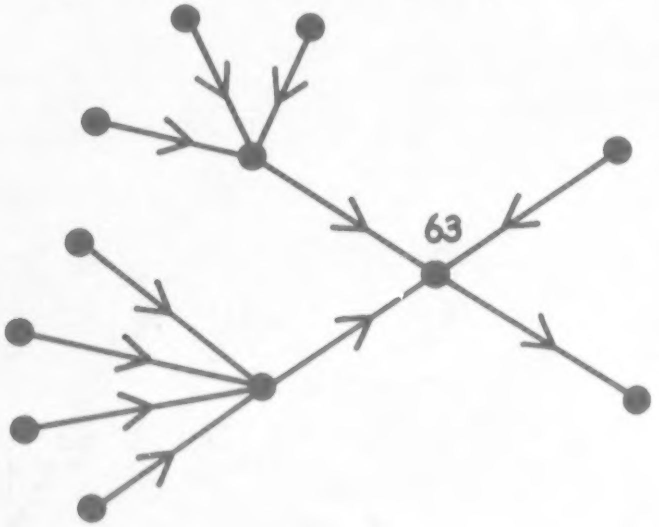
S: Nine. The center dot could be 1, 2, . . . , 9 since each of these numbers sends a valentine to 0.

Pages 13 and 14

Ask a student to read the pages aloud.

T: On page 14, label all the dots which are centers of flowers and some of the other dots.

As the students are working independently, draw this picture on the board.

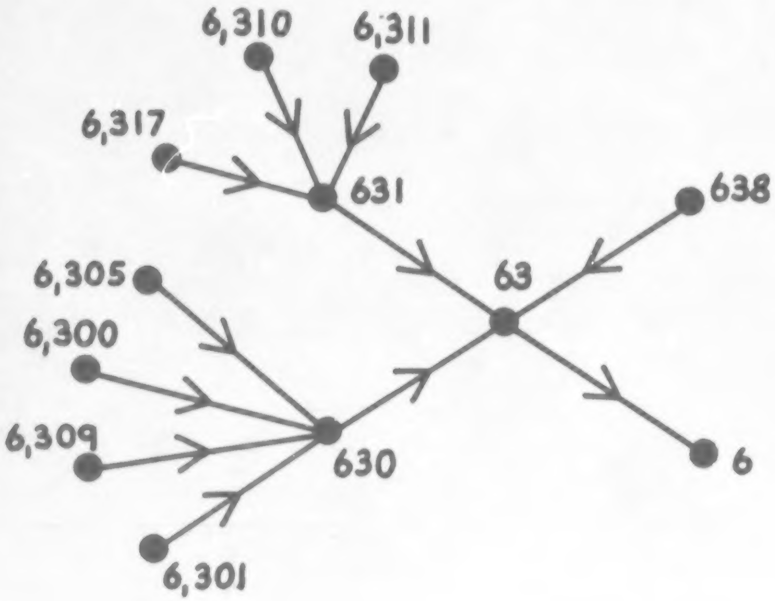


W 3

After a few minutes, direct the class' attention to the picture on the board.

T:	This is part of the picture on page 14. Who can label the dots?

Invite students to label the dots. The class should observe that many solutions are possible. One possible solution is shown below.

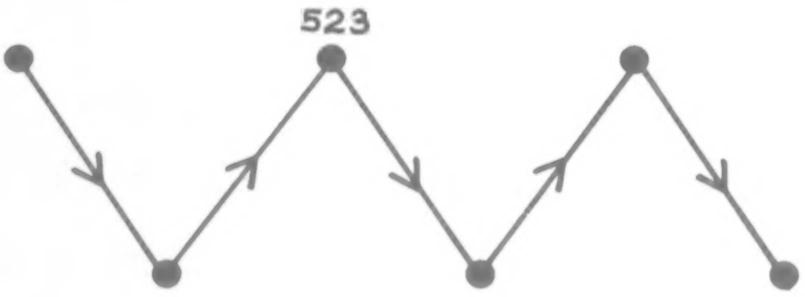

6,310
6,311
6,317
631
638
63
6,305
6,300
6,309
630
6
6,301

Pages 15 and 16

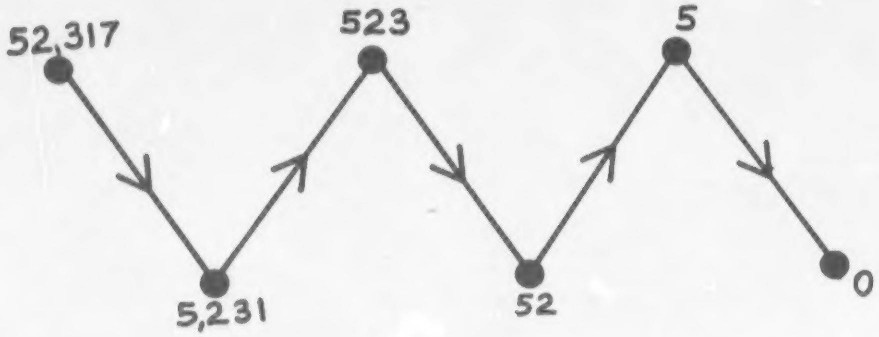
Read these pages aloud.

T:	Now close your storybooks. I will put another problem on the board.

Draw this arrow picture on the board.



Invite your students to label these dots. The three dots on the right must be labeled "52", "5", and "0" respectively. There are many possibilities for the two dots on the left. The picture below provides one possible solution.



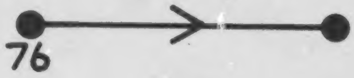
Collect the students' copies of the storybooks for use in Lesson 2.

Lesson 2

Distribute paper and colored pencils. Have copies of the storybook "A Valentine Mystery" ready, but do not distribute them until later. Ask a student to tell the part of the story read thus far.

Exercise 1

Draw this arrow picture on the board.



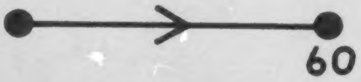
T: To whom does 76 send a valentine?

S: 7.

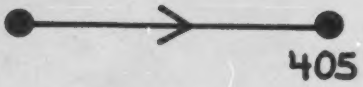
T: Does 7 receive other valentines?

S: Yes, 7 receives exactly ten valentines -- one each from 70, 71, 72, . . . , 78, and 79.

Similarly, ask who sends valentines to 60 and to 405.



(Answer: the whole numbers from 600 to 609)

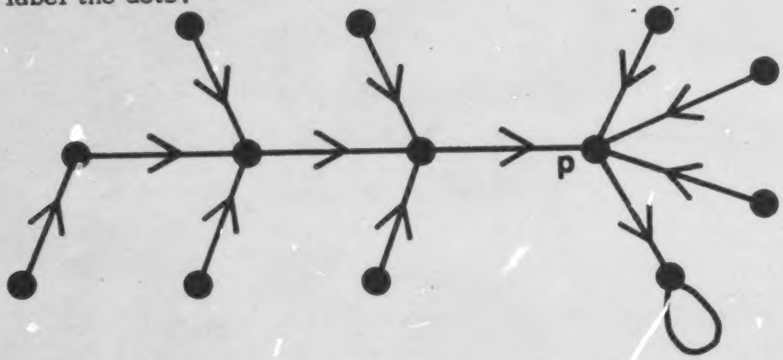


(Answer: the whole numbers from 4,050 to 4,059)

Repeat this exercise using other numbers until you feel that your students understand the rule for sending and receiving valentines.

Exercise 2

Draw the following arrow picture on the board. Ask your class to copy it and to label the dots.



Do not write the letter on the board. It is here just to make the description of the lesson easier to follow.

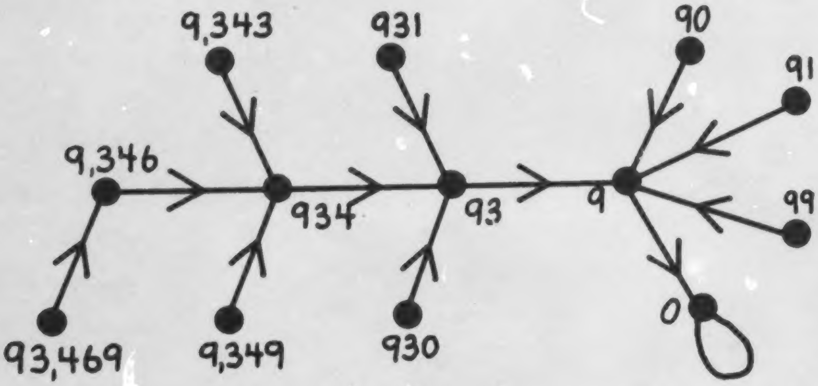
Let your students work on this problem independently for a few minutes, then solve it collectively.

T: Who can label one dot?

S: The dot with a loop must be 0 because 0 is the only number who sends a valentine to itself.

S (pointing to p): I put 9 here, but any number from 1 to 9 could be here because they all send valentines to 0.

Invite students to label the other dots on the board. The class should notice that many solutions are possible. The picture below gives one possible solution.



W 4

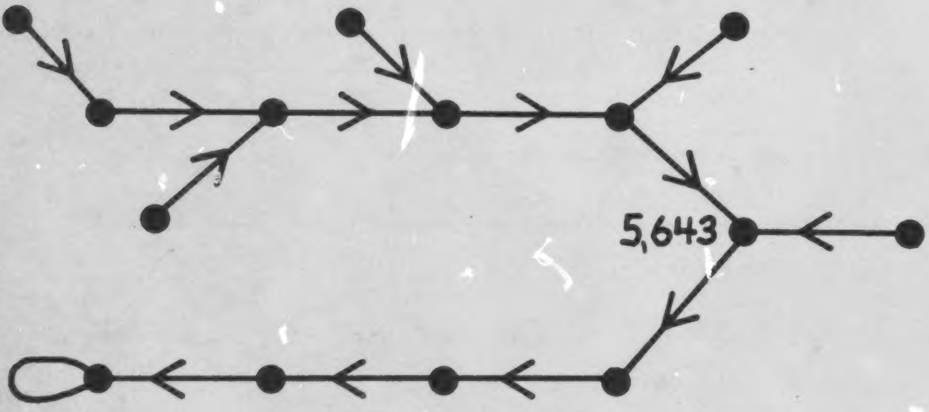
Distribute the students' copies of "A Valentine Mystery".

Pages 17 and 18

Ask a student to read these pages aloud.

T: Label all the center dots of flowers and some of the other dots on pages 17 and 18. Notice that one dot on page 18 is already labeled "5,643".

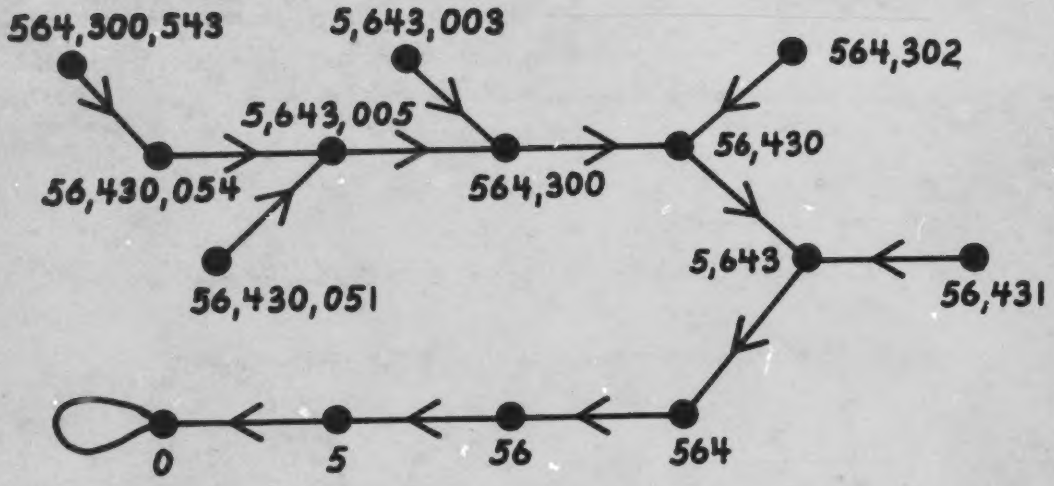
While students are working, draw this arrow picture on the board.



After several minutes, call the class' attention to the board.

T: This is part of the picture on pages 17-18. Who can label one of the dots?

Invite your students to label dots until they are all labeled. The numbers smaller than 5,643 must be 564, 56, 5, and 0. There are many possibilities for the other dots. Encourage your students to read the numbers as they write them in the picture. One possible solution is shown below.


564,300,543
5,643,003
564,302
5,643,005
56,430,054
564,300
56,430
56,430,051
5,643
56,431
0
5
56
564

Pages 19 - 22

T: Pages 19 and 20 give some hints on how to label the dots in the poster on pages 17 and 18. A full solution is on pages 21 and 22.

Allow some time for the students to read and study the pictures on these pages.

If you feel that most of your students have difficulty working with large numbers, you may wish to read the rest of the story as a class without stopping to do the work. In this case, suggest that interested students finish the work on their own.

W 4

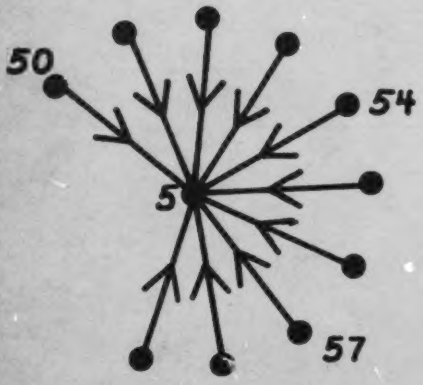
Otherwise, continue by reading and discussing pages 23 and 24.

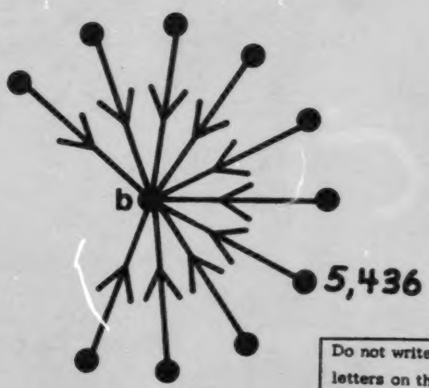
Pages 23 and 24

T: The flowers on pages 23 and 24 can be connected by red arrows. To help figure out where to draw the red arrows, label the center dot of each flower. I suggest that you begin on page 24 because the smaller numbers are there.

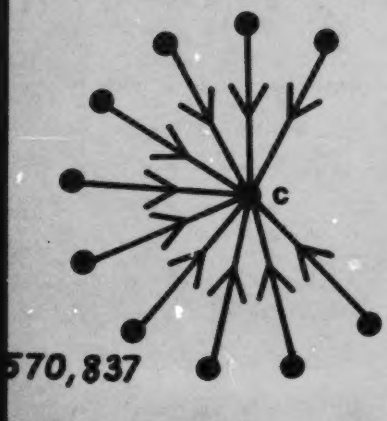
Let your class work on these pages for several minutes. Then tape Workbook Poster #1 for IG-IV on the board. Note: This poster is in two pieces and should be displayed to look exactly like pages 23 and 24 in the storybook.

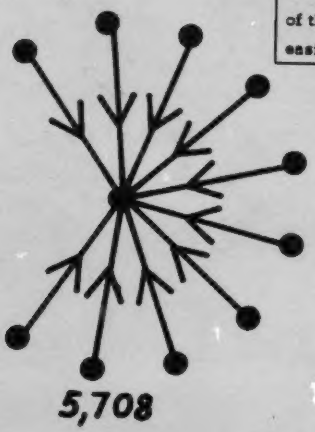
Call the students' attention to the upper half of the right side of the poster.





Do not write the letters on the poster. They are here just to make the description of the lesson easier to follow.





T: This poster is a copy of pages 23 and 24 of your storybook. Which number is this (point to b)?

S: 543, because 5,436 sends a valentine to 543.

T: Do you see a red arrow we could draw?

S: From 543 to 54.

Draw a red arrow from 543 to 54 on the poster.

T: Can someone label another center dot?

S (pointing to c): This is 57,083.

S: We can draw a red arrow from 57,083 to 5,708.

Continue in a similar manner until all the missing red arrows have been drawn. Use the completed picture on pages 25 and 26 of the storybook as your answer key. Labeling the center dots of the flowers aids in finding the red arrows. Encourage your students to both write and read the numbers as they are put in the picture.

Pages 25 - 28

After completing the poster, tell your class to look at the solution on pages 25 and 26 then to read pages 27 and 28.

Discuss the picture on pages 27 and 28.

T: Which number is in the center of the picture? (0)
Is the picture finished? (No)
Could I ever finish drawing the picture? (No, each number receives valentines from ten different numbers, and then each of those numbers receive ten more valentines and so on forever.)

Pages 29 and 30

Read and discuss these pages.

W 4

T: Why can this game be played by numbers but not by people?

S: There are a limited number of people in the world, but there are always more and larger numbers.

A
SELECTION
OF
PROBLEMS

#2

A SELECTION OF PROBLEMS #2

The problems in this workbook are in order of increasing difficulty. Approximately the first ten pages of the workbook have easy problems, the next ten to twelve pages have average problems, and the last ten pages have more challenging problems. It is suggested that all students work through as many pages as they can handle during the two lessons scheduled for the workbook. Not all students are expected to complete the entire workbook; some will succeed to do only the easy problems, many should finish about twenty pages, and only the faster students will reach the most challenging problems at the end of the booklet.

Many problems in the workbook were designed with the availability of hand-calculators in mind. If possible, provide all the students with hand-calculators or at least give students access to hand-calculators.* You will find that many students do not need or want to use calculators; however, some students still need the support of the calculator in thinking about these problems.

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* See Section 2(f) of the General Introduction for additional comments on the use of hand-calculators and on desirable characteristics of hand-calculators.

W 5 & 6

Lesson 1

Give each student a copy of the workbook "A Selection of Problems #2". A short comment about the instructions on page 2 may prove helpful to get the class started.

T: Turn to page 2. On this page, try to label some of the red arrows in two ways. For example, the first red arrow has two possible labels. It could be for +6 because 3 + 6 = 9, and it could be for ×3 because 3 × 3 = 9. You are required to label each arrow in only one way, but try to label some arrows in two ways.

Instruct the students to work on their own. If you observe many students struggling with the same problem (or part of a problem), you could have a collective discussion about that problem.

At the end of the lesson, collect the workbooks and have them ready for Lesson 2.

Lesson 2

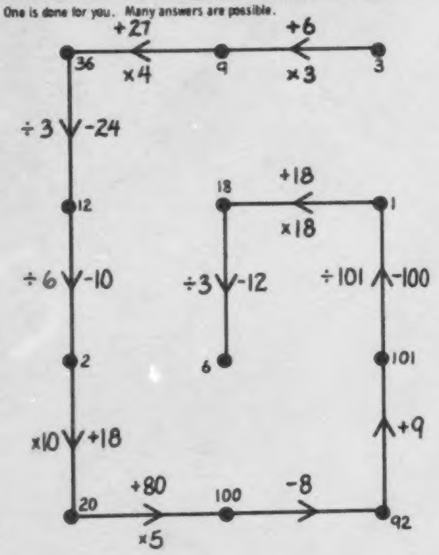
Return the students' copies of "A Selection of Problems #2". You may wish to discuss one or two pages that were difficult for most of the students. Ask the students to correct their mistakes, if any, and then to continue working in their workbooks.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion of that problem.

At the end of this lesson, collect the workbooks for your progress check. You may wish to let some students work further in their workbooks later during any free study time.

A SELECTION OF PROBLEMS #2

Label each red arrow. Try to find two ways to label some of the arrows.
One is done for you. Many answers are possible.


+27
×4
+6
×3
36
9
3
÷3
-24
12
18
+18
×18
1
÷6
-10
÷3
-12
÷101
-100
2
6
101
×10
+18
+9
20
+80
100
-8
92
×5

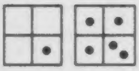
2

(Other solutions are possible.)

Kay is a secret number.

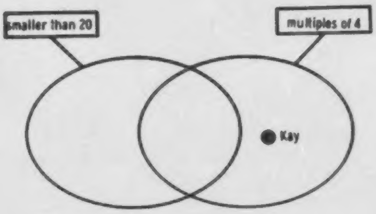
First clue

Kay can be shown on these Minicomputer boards by removing exactly one checker.



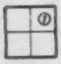
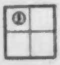
Kay could be 16, 18, 22, 24, or 25.

Second clue

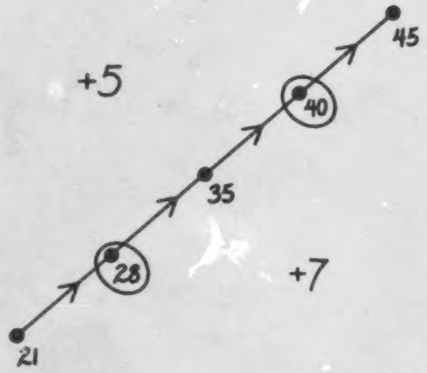

smaller than 20
multiples of 4
Kay

Kay is 24.

3

Luf =  – 28   Puf =  – 40

Luf and Puf are in this arrow picture.
Circle their dots. Label all the dots.

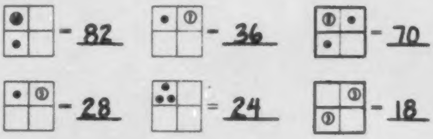

+5
+7
21
28
35
40
45

4

Tor is a secret number.

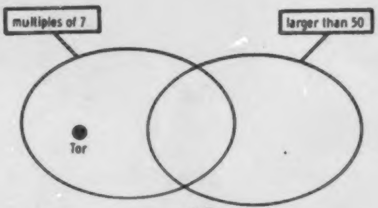
First clue

Tor is one of these numbers.



– 82
– 36
– 70
– 28
= 24
– 18

Second clue


multiples of 7
larger than 50
Tor

Tor is 28.

5

Put one of the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in each box to make the calculations correct. You may use a digit more than once.

Add

$$\begin{array}{r} 8\,2\,7\,4 \\ +\ \boxed{4}\,6\,\boxed{5} \\ \hline \boxed{8}\,7\,\boxed{3}\,9 \end{array}$$

Add

$$\begin{array}{r} \boxed{5}\,\boxed{7}\,3\,9 \\ +\ 2\,4\,\boxed{3} \\ \hline 5\,9\,\boxed{8}\,2 \end{array}$$

Add

$$\begin{array}{r} 63,709 \\ +\ 538 \\ \hline 64,247 \end{array}$$

Subtract

$$\begin{array}{r} \boxed{7}\,3\,\boxed{4}\,9 \\ -\ 1\,0\,3\,\boxed{3} \\ \hline 6\,\boxed{3}\,1\,6 \end{array}$$

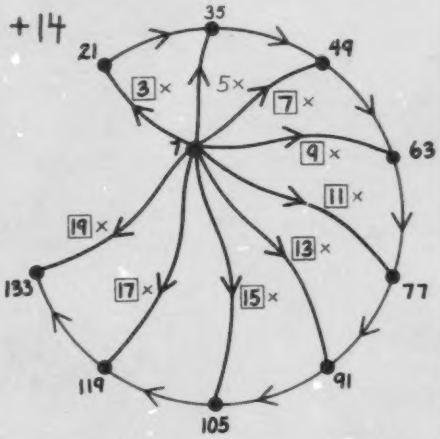
Subtract

$$\begin{array}{r} 3\,8\,7\,5\,\boxed{9} \\ -\ 2\,\boxed{4}\,\boxed{2}\,4 \\ \hline \boxed{3}\,\boxed{6}\,3\,3\,5 \end{array}$$

Subtract

$$\begin{array}{r} 40,729 \\ -\ 503 \\ \hline 40,226 \end{array}$$

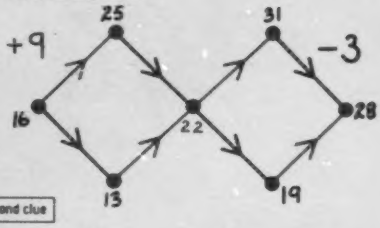
Label the dots and fill in the boxes for the red arrows.



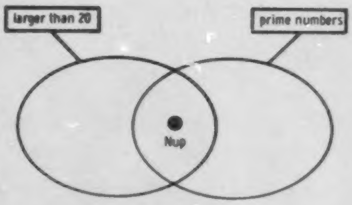
Nup is a secret number.

First clue

Nup is in this arrow picture.



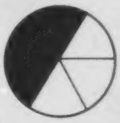
Second clue



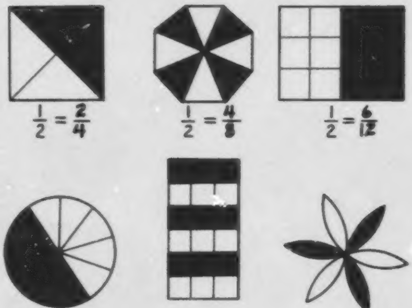
Nup is 31.

Color one-half of each shape red. In each case, write another name for $\frac{1}{2}$ as suggested by the picture.

EXAMPLE



$\frac{1}{2} = \frac{3}{6}$

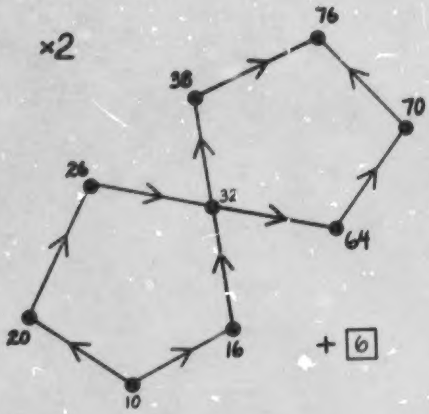


$\frac{1}{2} = \frac{2}{4}$ $\frac{1}{2} = \frac{4}{8}$ $\frac{1}{2} = \frac{6}{12}$

$\frac{1}{2} = \frac{5}{10}$ $\frac{1}{2} = \frac{9}{18}$ $\frac{1}{2} = \frac{3}{6}$

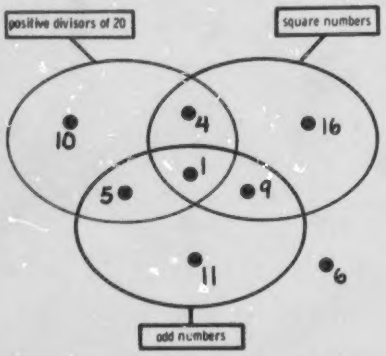
(Many other colorings are possible.)

Label the dots and fill in the box for the blue arrows.


×2
76
38
70
26
32
64
20
16
10
+ 6

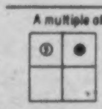
Label each dot in the string picture with one of these numbers.

1 4 5 6 9 10 11 16


positive divisors of 20
square numbers
10
4
16
1
5
9
11
6
odd numbers

Put each number on the Minicomputer by adding exactly one regular positive checker: ●. There may be more than one solution.

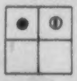
A multiple of 4



= 44

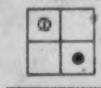
(48 is another solution.)

A multiple of 5



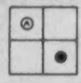
= 40

An odd number



= 57

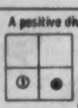
A number smaller than ⁻5



= ⁻7

(⁻6 is another solution.)

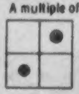
A positive divisor of 28



= 7

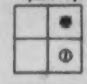
(14 is another solution.)

A multiple of 7



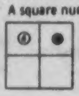
= 42

A positive prime number



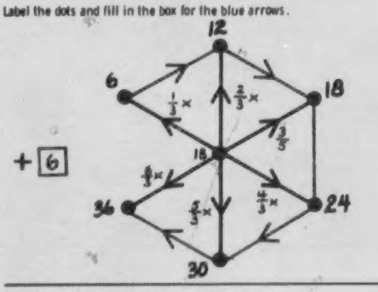
= 11

A square number

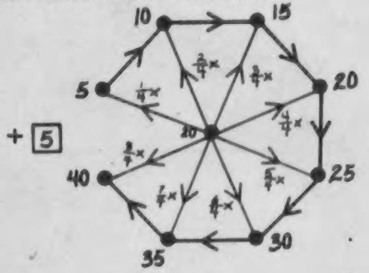


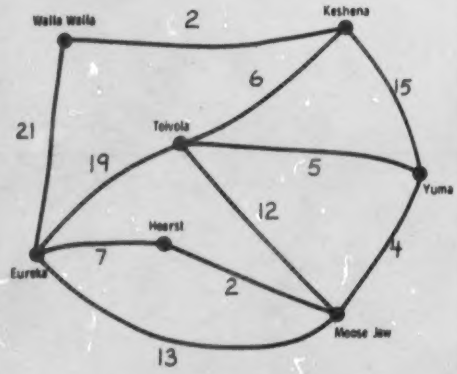
= 36

Label the dots and fill in the box for the blue arrows.


12
6
18
1/3 ×
2/3 ×
3/3
18
+ 6
6/3 ×
36
5/3 ×
4/3 ×
24
30

Label the dots and fill in the box for the red arrows.


10
15
2/4 ×
3/4 ×
5
1/4 ×
20
4/4 ×
20
+ 5
8/4 ×
40
5/4 ×
25
7/4 ×
6/4 ×
35
30


Walla Walla
Keshena
Toivola
Yuma
Hearst
Eureka
Moose Jaw
2
6
15
21
19
5
12
7
2
4
13

What is the length of the shortest road between Eureka and Keshena? 23

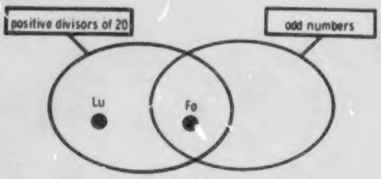
What is the length of the shortest road between Keshena and Moose Jaw? 15

What is the length of the shortest road between Eureka and Toivola? 18

14

Lu and Fo are secret numbers.

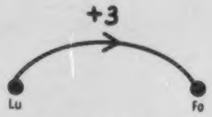
First clue


positive divisors of 20
odd numbers
Lu
Fo

Lu could be 2, 4, 10, or 20.

Fo could be 1 or 5.

Second clue



Lu is 2.

Fo is 5.

15

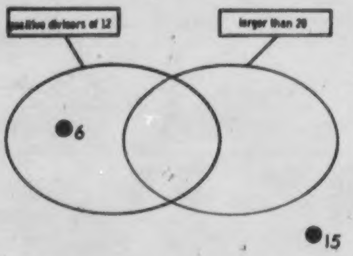
The red tag is one of these:

- multiples of 3
- multiples of 5
- odd numbers
- smaller than 50
- larger than 20
- positive divisors of 12
- positive divisors of 30

The blue tag is one of these:

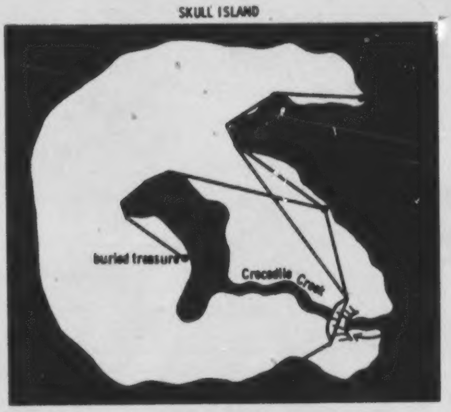
- multiples of 3
- multiples of 5
- odd numbers
- smaller than 50
- larger than 20
- positive divisor of 12
- positive divisors of 30

Label the strings.


positive divisors of 12
larger than 20
6
15

16

SKULL ISLAND


buried treasure
Crocodile Creek

Draw a zig-zag path as short as you can between the places listed below. You cannot go through any water.
Then measure in centimeters the length of the zig-zag path.

Fort Forlorn and sunken ship 7.2 cm
Lookout Point and Fort Forlorn 11.0 cm
Fort Forlorn and buried treasure 12.4 cm
Lookout Point and sunken ship 17.9 cm

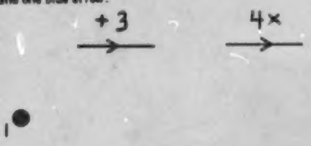
17

Students' answers should agree with the paths they draw. When an answer is larger than the solution indicated here, you could suggest that a shorter path is possible, but do not insist on exactly the same answers.

Xaf is a secret number.

First clue

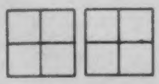
Xaf is the ending number of a road starting at 1 and using exactly two red arrows and one blue arrow.



Xaf could be 10 or 19 or 28.

Second clue

Xaf cannot be put on the Minicomputer with exactly two regular checkers.



Who is Xaf 19 ?

Put one of the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in each box to make the calculations correct. You may use a digit more than once.

Add

$$\begin{array}{r} 5\boxed{3}7 \\ 4\boxed{7} \\ +1\boxed{0}24 \\ \hline \boxed{1}608 \end{array}$$

Add

$$\begin{array}{r} 67\boxed{9} \\ \boxed{3}\boxed{0}39 \\ +\ 3\boxed{8}9 \\ \hline 4107 \end{array}$$

Subtract

$$\begin{array}{r} \boxed{1}\boxed{2}73 \\ -\ 7\boxed{2}8 \\ \hline 54\boxed{5} \end{array}$$

Subtract

$$\begin{array}{r} 509 \\ -\boxed{3}74 \\ \hline 1\boxed{3}\boxed{5} \end{array}$$

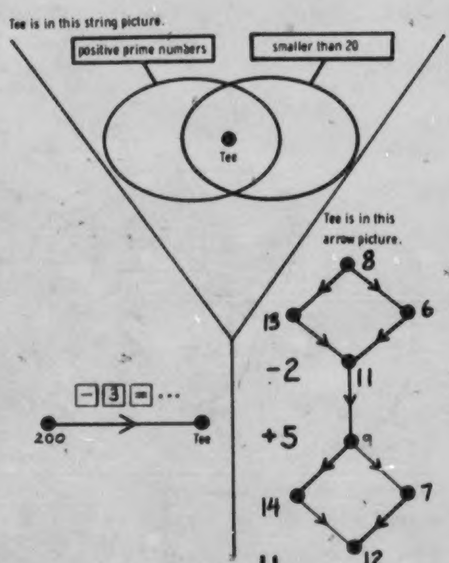
Multiply

$$\begin{array}{r} \boxed{1}6\boxed{5} \\ \times\ \ 7 \\ \hline 11\boxed{5}5 \end{array}$$

Multiply

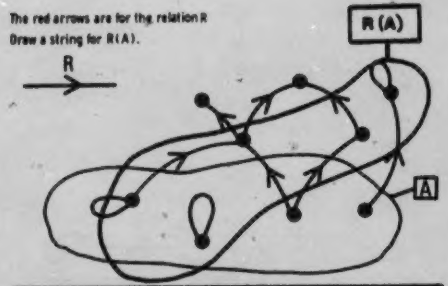
$$\begin{array}{r} \boxed{5}83 \\ \times\ \ \boxed{6} \\ \hline 3\boxed{4}\boxed{9}8 \end{array}$$

Tee is in this string picture.

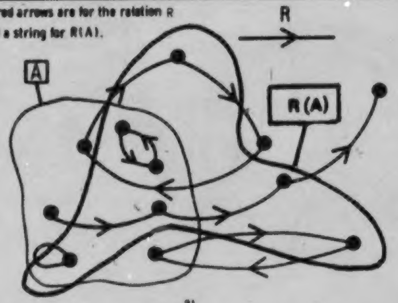

positive prime numbers
smaller than 20
Tee
Tee is in this arrow picture.
8
6
15
-2
11
+5
9
14
7
12
−3=…
200
Tee

Who is Tee? 11

The red arrows are for the relation R
Draw a string for R(A).



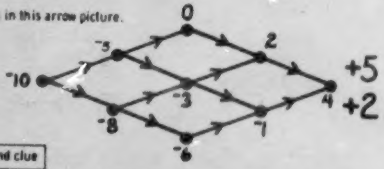
The red arrows are for the relation R
Draw a string for R(A).



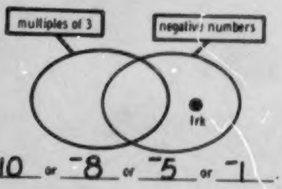
Irk is a secret number.

First clue

Irk is in this arrow picture.


0
⁻5
2
⁻10
⁻3
4
+5
+2
⁻8
⁻1
⁻6

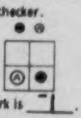
Second clue


multiples of 3
negative numbers
Irk

Irk could be ⁻10 or ⁻8 or ⁻5 or ⁻1.

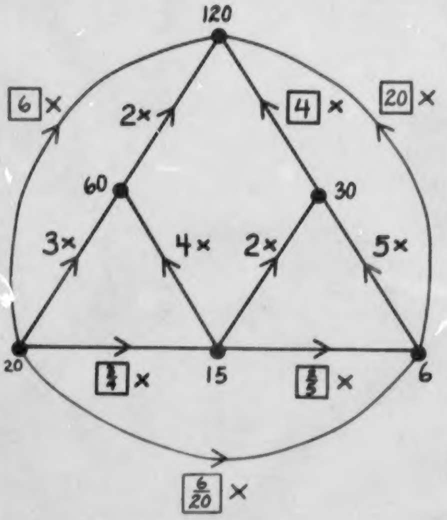
Third clue

Irk can be put on the ones' board of the Minicomputer with exactly one negative checker and one regular checker.

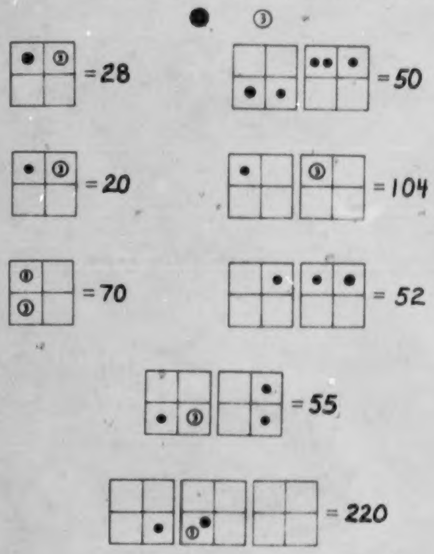


Irk is ⁻1.

Label the dots and fill in the boxes for the arrows.

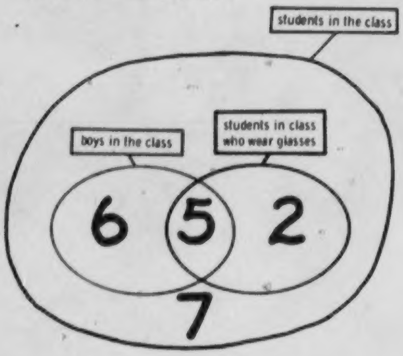

120
6 ×
2×
4 ×
20 ×
60
30
3×
4×
2×
5×
20
$\frac{3}{4}$ ×
15
$\frac{2}{5}$ ×
6
$\frac{6}{20}$ ×

Put each number on the Minicomputer by adding exactly one of these checkers.


= 28
= 50
= 20
= 104
= 70
= 52
= 55
= 220

There are 20 students in the class.

6 boys in the class do not wear glasses.
11 students in the class are boys.
7 students in the class wear glasses.


students in the class
boys in the class
students in class who wear glasses
6
5
2
7

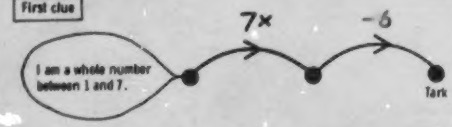
How many boys in the class wear glasses? 5

How many girls are there in the class? 9

How many girls in the class do not wear glasses? 7

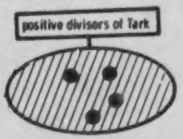
Tark is a secret WHOLE number.

First clue


7×
-6
I am a whole number between 1 and 7.
Tark

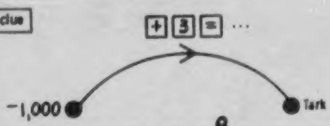
Tark could be 8, 15, 22, 29, or 36.

Second clue


positive divisors of Tark

Tark could be 8, 15, or 22.

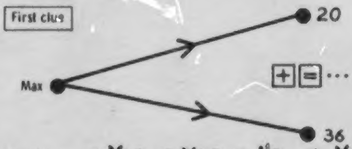
Third clue



Who is Tark? 8

Max is a secret number.

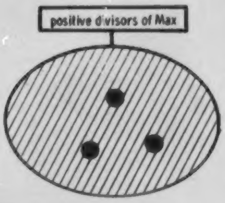
First clue



Could Max be 1? Yes ; 2? Yes ; 3? No ; 4? Yes ;
5? No ; 6? No ; 0.1? Yes ; 0.3? No ; 0.5? Yes ;
0.25? Yes

Second clue

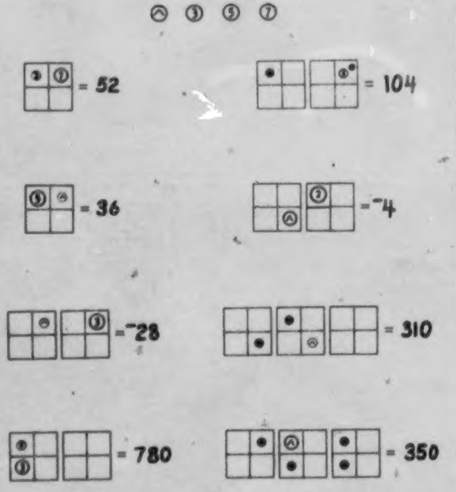
Max is a whole number.[5]


positive divisors of Max

Max is 4.

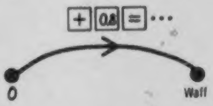
[5] A student may respond "No" to all the non-integer decimals in the first clue because the second clue states "Max is a whole number." Accept this interpretation as correct if it occurs.

Put each number on the Minicomputer by adding exactly one of these checkers.


= 52
= 104
= 36
= ⁻4
= ⁻28
= 310
= 780
= 350

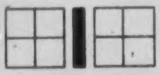
Waff is a secret number.

First clue



Second clue

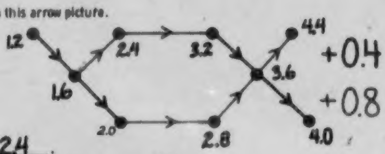
Waff can be put on this Minicomputer with exactly two regular checkers, one on each board.



Waff could be 2.4, 4.8, or 8.8.

Third clue

Waff is in this arrow picture.


1.2
1.6
2.4
3.2
3.6
4.4
2.0
2.8
4.0
+0.4
+0.8

Waff is 2.4.

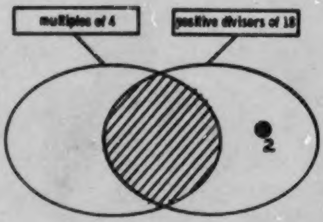
The red tag is one of these:

- smaller than 20
- larger than ⁻10
- multiples of 2
- multiples of 4
- positive divisors of 18
- positive divisors of 20

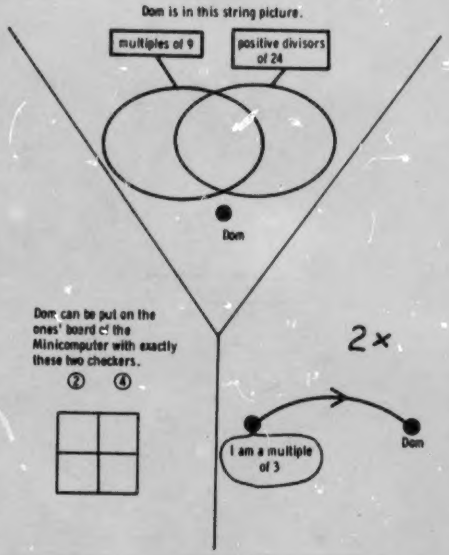
The blue tag is one of these:

- smaller than 20
- larger than ⁻10
- multiples of 2
- multiples of 4
- positive divisors of 18
- positive divisors of 20

Label the strings.


multiples of 4
positive divisors of 18
2

Dom is in this string picture.


multiples of 9
positive divisors of 24
Dom
2×
I am a multiple of 3
Dom

Dom can be put on the ones' board of the Minicomputer with exactly these two checkers.

② ④

Who is Dom? 48

Write a name for each number using exactly four "4" 's and no other digits. You may use the following symbols as often as you wish.

+ − ( ) × ÷

The number 6 is done for you.

(4+4)−(4+4) = 0

(4+4)÷(4+4) = 1

(4÷4)+(4÷4) = 2

((4+4)+4)÷4 = 3

((4−4)×4)+4 = 4

((4×4)+4)÷4 = 5

((4+4)÷4)+4 = 6

(4+4)−(4÷4) = 7

((4×4)−4)−4 = 8

(4×4)−(4−4) = 16

((4×4)−4)×4 = 48

(4×4)+(4÷4) = 17

((4×4)×4)−4 = 60

((4×4)+4)×4 = 80

(Many solutions are possible.)

A
SELECTION
OF
PROBLEMS

#3

A SELECTION OF PROBLEMS #3

The problems in this workbook are in order of increasing difficulty. Approximately the first ten pages of the workbook have easy problems, the next ten to twelve pages have average problems, and the last ten pages have more challenging problems. It is suggested that all students work through as many pages as they can handle during the two lessons scheduled for the workbook. Not all students are expected to complete the entire workbook; some will succeed to do only the easy problems, many should finish about twenty pages, and only the faster students will reach the most challenging problems at the end of the booklet.

Many problems in the workbook were designed with the availability of hand-calculators in mind. If possible, provide all the students with hand-calculators or at least give students access to hand-calculators.[*] You will find that many students do not need or want to use calculators; however, some students still need the support of the calculator in thinking about these problems.

A brief collective activity is suggested to begin each of the lessons in this workbook. These should be short warm-up activities which serve to present a new type of problem introduced in this workbook.

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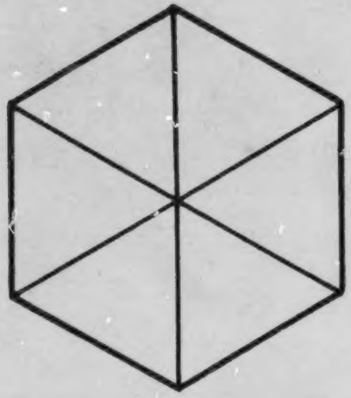
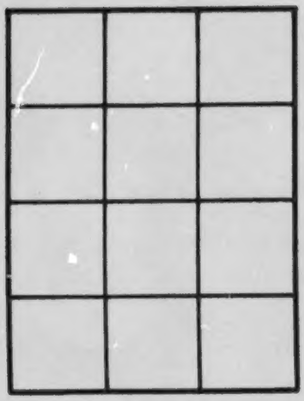
[*] See Section 2(f) of the General Introduction for additional comments on the use of hand-calculators and on desirable characteristics of hand-calculators.

W 7

Lesson 1

Draw these two shapes on the board.

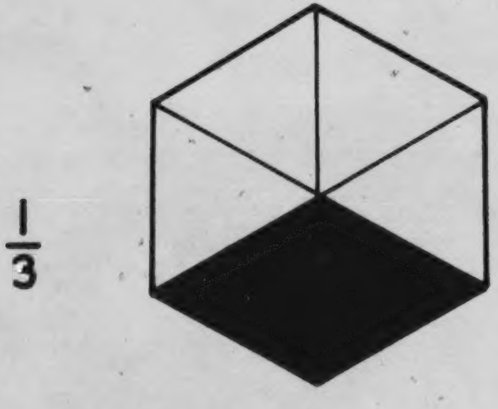




Point to the hexagon.

T: Who can shade one-third of this shape in red?

Invite a student to do the shading with red chalk. Many correct solutions are possible. For example, a student might suggest this shading.



T: Who can convince me that one-third of the shape is shaded red?

S: Two of the six triangles in the picture are shaded and 2 is one-third of 6.

S: You can tell by looking at the picture. One of the three triangles left of the center line is shaded and also one of three triangles right of the center line is shaded.

T: Is there another way to shade one-third of this shape?

S: You could shade any two of the six triangles.

NOTE: Students might also suggest more esoteric solutions such as dividing each triangle in thirds and shading one-third of each. Focus the class' attention on the idea of shading two of the six triangles.

T: Does this picture suggest another name for $\frac{1}{3}$?

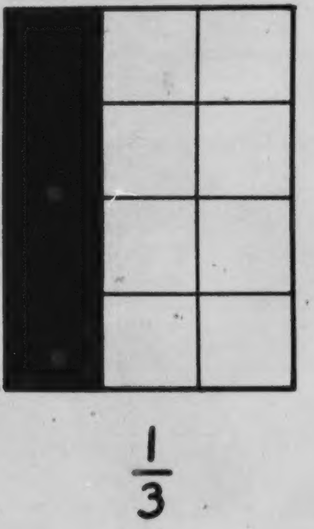
S: "$\frac{2}{6}$", because shading two of the six triangles is the same as shading one-third of the shape.

Near the hexagon, complete the equation: $\frac{1}{3} = \frac{2}{6}$

Point to the rectangle drawn on the board.

T: Who can shade one-third of this rectangle?

Invite a student to do the shading, again with red chalk. Many correct solutions are possible; for example, a student might suggest this shading.



W 7

T: Is one-third of this shape shaded red?

S: Yes, because one of the three columns is shaded.

S: Yes, because 4 is one-third of 12, and four of the twelve squares are shaded.

T: Is there another way to shade one-third of this shape?

S: You could shade any one of the three columns.

S: You could shade any four of the twelve squares.

T: Does this picture suggest another name for $\frac{1}{3}$?

S: "$\frac{4}{12}$".

Near the rectangle, complete the equation: $\frac{1}{3} = \frac{4}{12}$

Give each student a copy of the workbook "A Selection of Problems #3" and let them work on their own.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the end of the lesson, collect the workbooks and have them ready to return for Lesson 2.

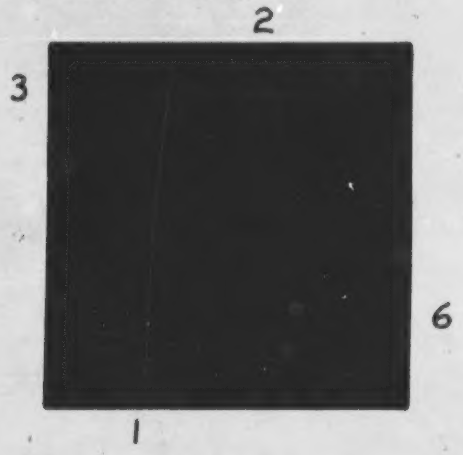
Lesson 2

Before class, carefully draw this picture on the board.

Point to the blue rectangle in the lower left part of the figure.

T: How many pieces of this size could fit into each of the blue regions?

Invite students to explain their answers and follow the students' suggestions as they present their methods of solution. For example, a student may suggest dividing each of the larger blue regions into smaller parts. When the class agrees on how many small pieces fit into each blue region, write the appropriate number in blue near each region.



T: Altogether, how many of the small pieces fit into the blue regions?

S: 12, because 1 + 2 + 3 + 6 = 12.

Erase the other blue numerals and write a large "12" in blue near the picture. Point to the red rectangle in the lower left corner.

T: How many of these small pieces could fit into the red region?

Encourage the class to solve this problem in a similar manner. The next illustration shows the final picture with each of the red regions subdivided.

T: What part of the square is shaded red?

S: $\frac{12}{24}$ or $\frac{1}{2}$.

Return the students' copies of "A Selection of Problems #3". Ask the students to correct their mistakes, if any, and then to continue working in their workbooks.

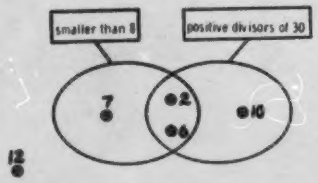
If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion of the problem.

At the end of this lesson, collect the workbooks for your progress check. You may wish to let some students work further in their workbooks later during any free study time.

A SELECTION OF PROBLEMS #3

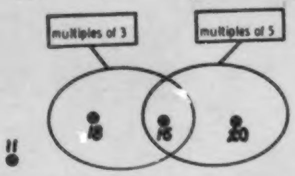
Put each of these numbers in the string picture.

2 6 7 10 12

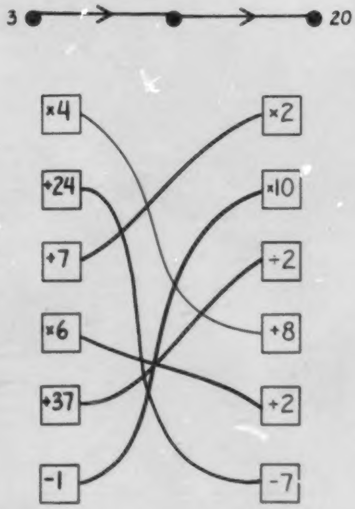

smaller than 8
positive divisors of 30
7
2
6
10
12

Put each of these numbers in the string picture.

11 15 18 20


multiples of 3
multiples of 5
18
15
20
11

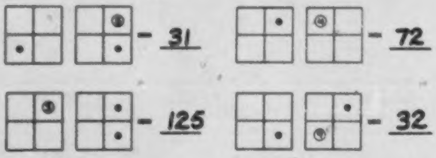
Pair the tags. One is done for you.


3
20
×4
+24
+7
×6
+37
-1
×2
×10
÷2
+8
+2
-7

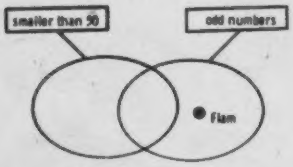
Flam is a secret number.

First clue

Flam is one of these numbers.

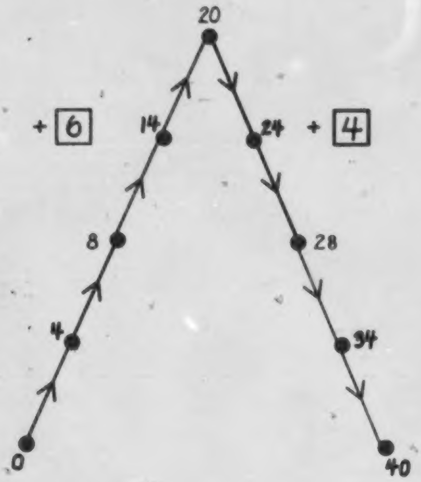


Second clue

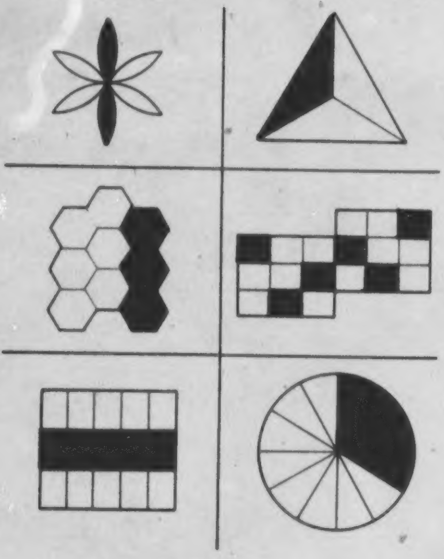

smaller than 50
odd numbers
Flam

Who is Flam? 125

Fill in the boxes for the arrows.
Label the dots.

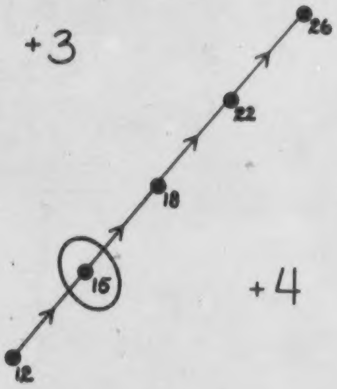

20
+ 6
14
24
+ 4
8
28
4
34
0
40

Color one-third of each shape blue.



(Many solutions are possible.)

15 is the only odd number in this picture.
Find the dot for 15. Then label all the dots.



Put one of the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in each box to make the calculations correct. You may use a digit more than once.

Add

$$\begin{array}{r} 5\,\boxed{6}\,2 \\ +\,4\,8\,3\,\boxed{0} \\ \hline \boxed{5}\,\boxed{3}\,8\,2 \end{array}$$

Add

$$\begin{array}{r} \boxed{6}\,2\,6 \\ \boxed{3}\,6 \\ +\,3\,1\,6 \\ \hline 9\,7\,\boxed{8} \end{array}$$

Add

$$\begin{array}{r} 3\,4\,7 \\ 7\,8 \\ +\,6{,}4\,0\,7 \\ \hline 6{,}8\,3\,2 \end{array}$$

Subtract

$$\begin{array}{r} 8\,\boxed{6}\,5\,2 \\ -\,\boxed{3}\,0\,\boxed{3}\,6 \\ \hline 5\,6\,1\,\boxed{6} \end{array}$$

Subtract

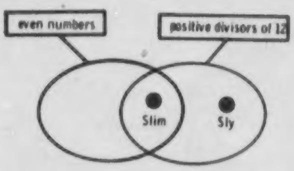
$$\begin{array}{r} 2\,\boxed{7}\,7\,4\,4 \\ -\,4\,\boxed{2}\,8\,\boxed{0} \\ \hline \boxed{2}\,3\,4\,\boxed{6}\,4 \end{array}$$

Subtract

$$\begin{array}{r} 6\,1{,}3\,7\,2 \\ -\,9\,2\,4 \\ \hline 6\,0{,}4\,4\,8 \end{array}$$

Sly and Slim are secret numbers.

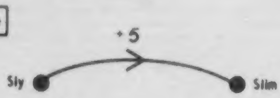
First clue



Slim could be 2, 4, 6, or 12.

Sly could be 1 or 3.

Second clue



Sly is 1.

Slim is 6.

How many squares of this size, ■, fit into the red region? 13

How many squares of this size, ■, fit into the blue region? 11

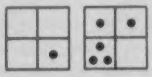
How many rectangles of this size, ■, fit into the red region? 14

How many rectangles of this size, ■, fit into the blue region? 10

Fro is a secret number.

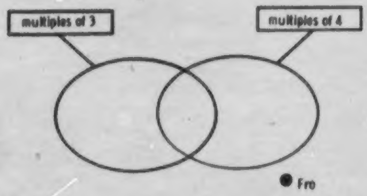
First clue

Fro can be shown on this Minicomputer by removing exactly one checker.



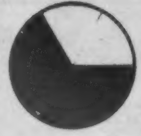
Fro could be 18, 20, 24, or 26.

Second clue

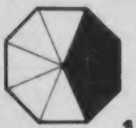

multiples of 3
multiples of 4
Fro

Fro is 26.

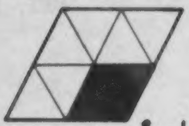
Write a fraction to indicate what part of the shape is colored blue. One is done for you.



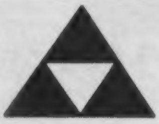
$\frac{4}{6}$ or $\frac{2}{3}$



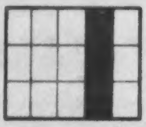
$\frac{3}{8}$



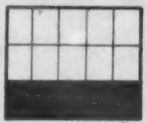
$\frac{2}{8}$ or $\frac{1}{4}$



$\frac{3}{4}$



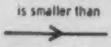
$\frac{3}{15}$ or $\frac{1}{5}$

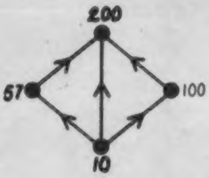


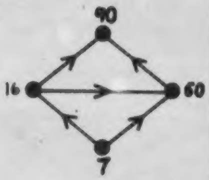
$\frac{5}{15}$ or $\frac{1}{3}$

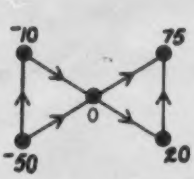
(The fractions given here are suggested by the pictures; however, other correct names could be given.)

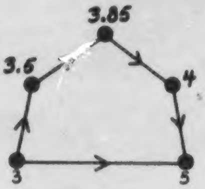
Label the dots. Many answers are possible.


is smaller than


200
57
100
10

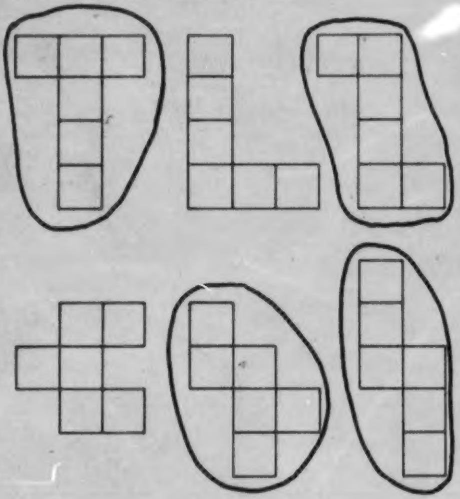






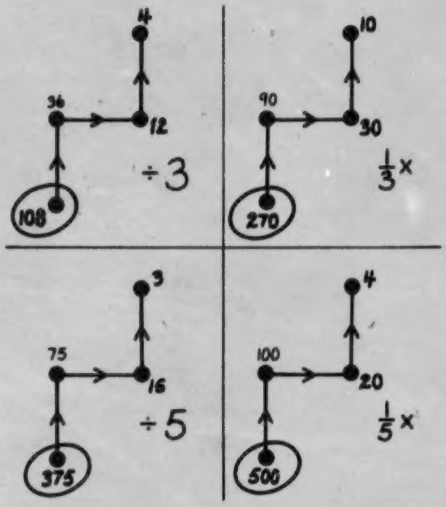
(Many solutions are possible.)

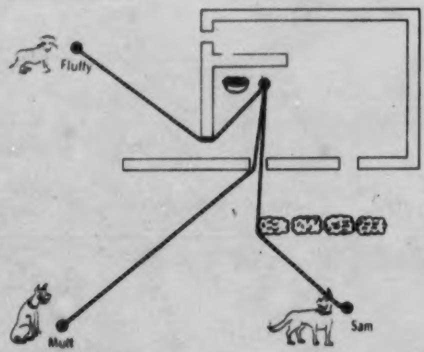
Circle each shape that could be folded into a cube.



You should have circled four shapes.

Label the dots. Circle the largest number in each arrow picture.





Draw a zig-zag path from each dog to the dish of food. Make the paths as short as you can. Measure the length (in centimeters) of each zig-zag.

Fluffy to food 9.9 cm[a]
Mutt to food 13.2 cm
Sam to food 10.4 cm

Which dog has the shortest path to the food? Fluffy

[a] **Students' answers should agree with the paths they draw. When an answer is larger than the solution indicated here, you could suggest that a shorter path is possible, but do not insist on exactly the same answers.**

Put one of the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in each box to make the calculations correct. You may use a digit more than once.

Subtract

$$\begin{array}{r} 2\,6\,\boxed{7}\,4 \\ -2\,\boxed{0}\,1\,8 \\ \hline 6\,5\,\boxed{6} \end{array}$$

Subtract

$$\begin{array}{r} 6\,\boxed{4}\,7\,3\,5 \\ -\quad 3\,\boxed{7}\,8 \\ \hline \boxed{6}\,4\,\boxed{3}\,5\,\boxed{7} \end{array}$$

Subtract

$$\begin{array}{r} 54{,}703 \\ -\,6{,}277 \\ \hline 48{,}426 \end{array}$$

Multiply

$$\begin{array}{r} \boxed{4}\,7\,\boxed{6} \\ \times\quad 9 \\ \hline 4\,2\,\boxed{8}\,4 \end{array}$$

Multiply

$$\begin{array}{r} \boxed{6}\,7\,0 \\ \times\quad \boxed{6} \\ \hline 4\,0\,2\,\boxed{0} \end{array}$$

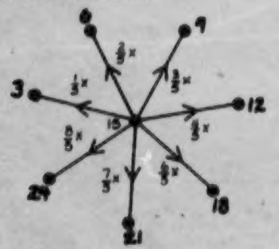
Multiply

$$\begin{array}{r} \boxed{3}\,5\,2 \\ \times\quad \boxed{8} \\ \hline 2\,8\,1\,6 \end{array}$$

Frik is a secret number.

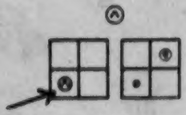
First clue

Frik is in this arrow picture. Label the dots.


6
9
3
12
15
24
21
18
2/5 ×
3/5 ×
1/5 ×
4/5 ×
8/5 ×
7/5 ×
6/5 ×

Second clue

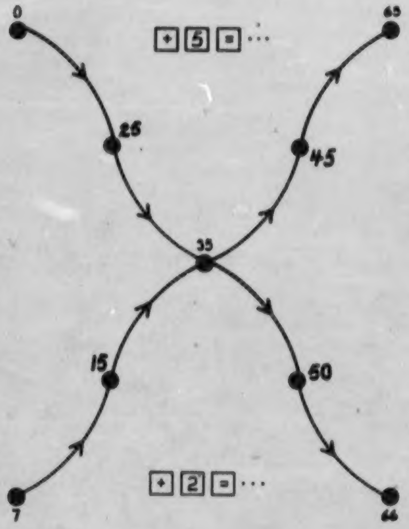
Frik can be put on this Minicomputer by adding exactly one negative checker.



Who is Frik? 18

18

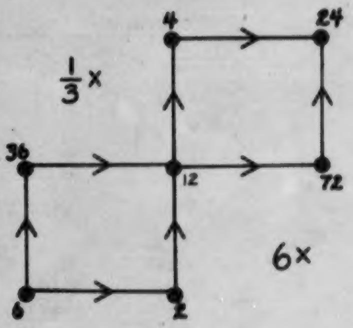
Label the dots. Many answers are possible.


0
65
+5=...
25
45
35
15
50
+2=...
7
64

19

(Many solutions are possible.)

Label the dots.


4
24
1/3 ×
36
12
72
6×
6
2

Pip and Tip are the only multiples of 9 in the arrow picture.
Pip is smaller than Tip.

Who is Pip? 36

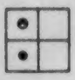
Who is Tip? 72

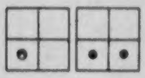
20

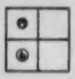
Kuth is a secret number.

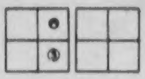
First clue

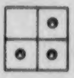
Kuth is one of these numbers.

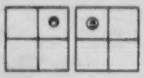

= ⁻6


= ⁻17

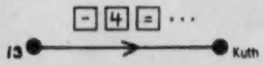

= 4


= ⁻10


= ⁻7


= 0

Second clue



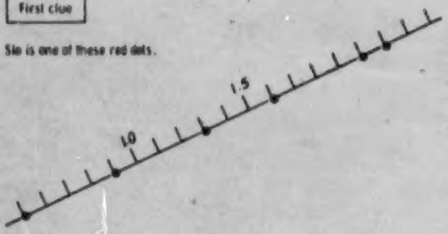
Who is Kuth? ⁻7

21

Slo is a secret number.

First clue

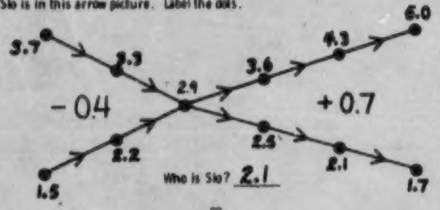
Slo is one of these red dots.



Slo could be 0.5, 0.9, 1.3, 1.6, 2.0, or 2.1.

Second clue

Slo is in this arrow picture. Label the dots.


3.7
3.3
2.9
3.6
4.3
6.0
-0.4
+0.7
1.5
2.2
2.5
2.1
1.7

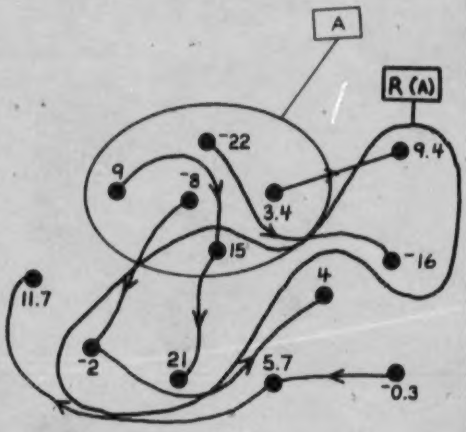
Who is Slo? 2.1

22

Draw as many red arrows in this picture as you can.

Draw a string for R(A).

R = +6

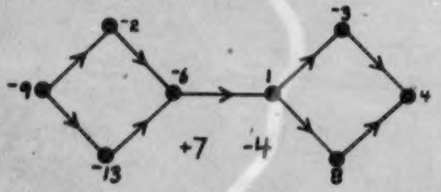

A
R (A)
$^{-}22$
9
$^{-}8$
3.4
9.4
15
$^{-}16$
11.7
4
$^{-}2$
21
5.7
$^{-}0.3$

23

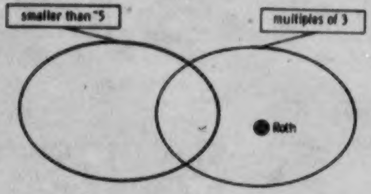
Roth is a secret number.

First clue

Roth is in this arrow picture.


$^{-}9$
$^{-}2$
$^{-}6$
$^{-}13$
1
$^{-}3$
4
8
+7
-4

Second clue


smaller than $^{-}5$
multiples of 3
Roth

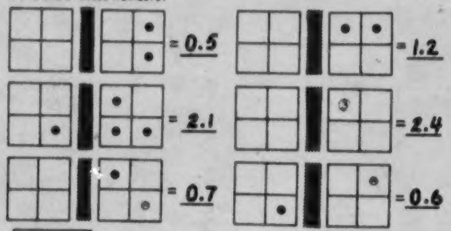
Roth is $^{-}3$.

24

Klo is a secret number.

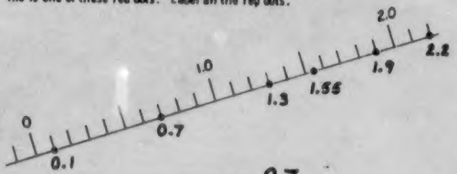
First clue

Klo is one of these numbers.


= 0.5
= 1.2
= 2.1
= 2.4
= 0.7
= 0.6

Second clue

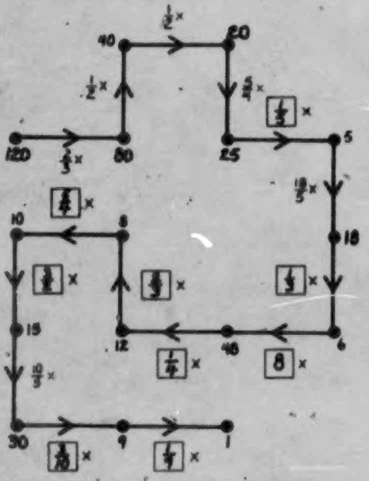
Klo is one of these red dots. Label all the red dots.


0
1.0
2.0
0.1
0.7
1.3
1.55
1.9
2.2

Klo is 0.7.

25

Label the dots and fill in the box for each red arrow.



$\frac{1}{2}\times$
40
20
$\frac{1}{2}\times$
$\frac{5}{4}\times$
$\boxed{\frac{1}{5}}\times$
120
$\frac{2}{3}\times$
80
25
5
$\frac{18}{5}\times$
$\boxed{\frac{5}{4}}\times$
10
8
18
$\boxed{\frac{3}{2}}\times$
$\boxed{\frac{2}{3}}\times$
$\boxed{\frac{1}{3}}\times$
15
12
48
6
$\boxed{\frac{1}{4}}\times$
$\boxed{8}\times$
$\frac{10}{3}\times$
30
9
1
$\boxed{\frac{3}{10}}\times$
$\boxed{\frac{1}{9}}\times$

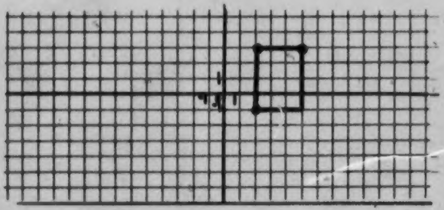
26

(Other correct labels for the red arrows could be given; e.g., $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12} = \ldots .$)

Three corners of a rectangle are at (2, 3), (2, ⁻1), and (5, 3).

Draw dots at these three corners and then draw the rectangle.

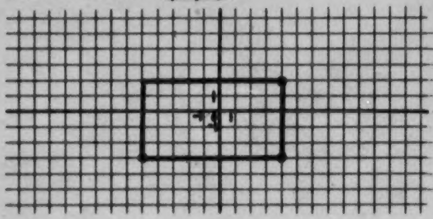
Where is the fourth corner? (5,⁻1)



Three corners of a rectangle are at (⁻5, ⁻3), (4, 2), and (4, ⁻3).

Draw dots at these three corners and then draw the rectangle.

Where is the fourth corner? (⁻5,2)

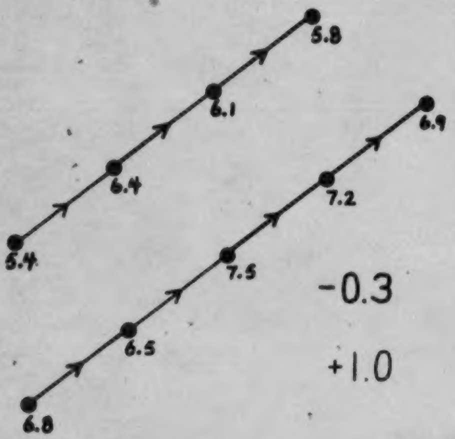


27

Label each dot in the arrow picture with one of these numbers.

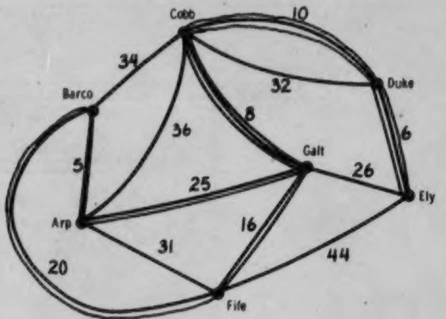
5.4 5.8 6.1 6.4

6.5 6.8 6.9 7.2 7.5



5.8
6.1
6.9
6.4
7.2
5.4
7.5
-0.3
6.5
+1.0
6.8

28



Cobb
10
34
Barco
32
Duke
36
8
6
Galt
26
5
25
Ely
Arp
16
31
44
20
Fife

The distance between two cities is the length of the shortest road. What is the distance from Arp to Duke? 43

List all the cities whose distance from Fife is less than 30. Arp, Barco, Cobb, and Galt

Draw in blue a journey of length less than 120 which starts in Cobb, visits each city at least once, and then ends in Cobb.

29

(The journey indicated above has length 114 and uses some paths more than once; for example, the path of length 6 is followed both going to Ely and returning from Ely. Other solutions are possible.)

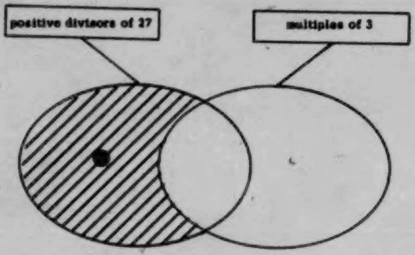
The red tag is one of these:

- multiples of 2
- multiples of 3
- larger than 20
- smaller than 20
- positive divisors of 18
- positive divisors of 20
- positive divisors of 27

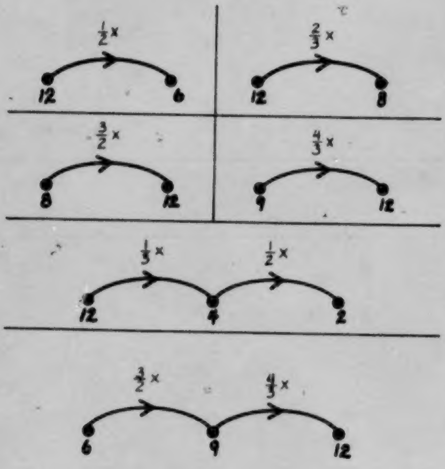
The blue tag is one of these:

- multiples of 2
- multiples of 3
- odd numbers
- larger than 5
- smaller than 5
- positive divisors of 20
- positive divisors of 27

Label the strings.


positive divisors of 27
multiples of 3

12 is the largest number in each arrow picture. Find 12 in each picture. Label all the dots.


$\frac{1}{2}\times$
12
6
$\frac{2}{3}\times$
12
8
$\frac{3}{2}\times$
8
12
$\frac{4}{3}\times$
9
12
$\frac{1}{3}\times$
$\frac{1}{2}\times$
12
4
2
$\frac{3}{2}\times$
$\frac{4}{3}\times$
6
9
12

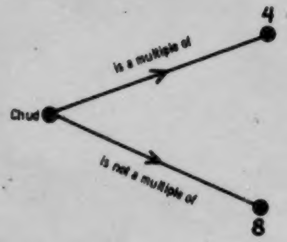
Chud is a secret number.

First clue

Chud is a four digit number with exactly two 5's, one 2, and one 8. For example, Chud could be 5,285.

Chud could be 5528, 5582, 5258, 5852, 5285, 5825, 2558, 8552, 2585, 8525, 2855, or 8255.

Second clue


4
is a multiple of
Chud
is not a multiple of
8

Who is Chud? 5852.

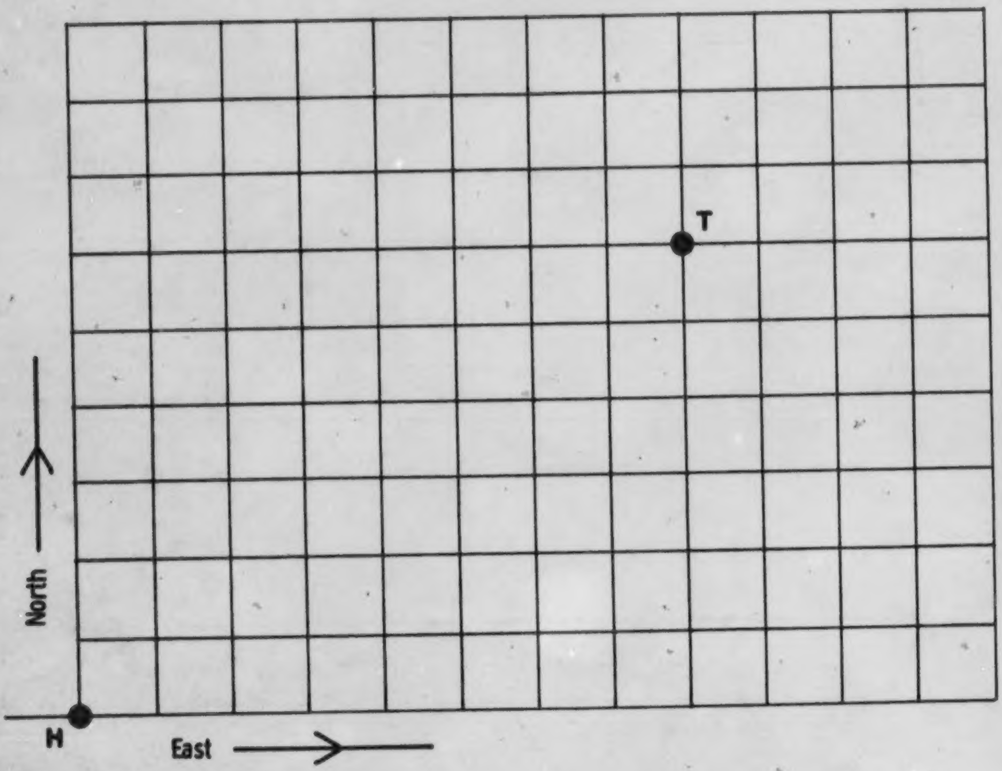
THE HIDDEN TREASURE

THE HIDDEN TREASURE

Two lessons are based on this storybook about a detective, Spick, who tries to solve a mystery about stolen diamonds. In each lesson, the teacher presents a numerical problem concerning Spick's investigation and the students find the solution. The storybooks are distributed to the class during the second lesson only after both problems have been solved. By reading this enjoyable story about Spick, the students review the methods that are used in class to solve the two problems.

Lesson 1

Distribute Worksheet N63 (Grid) and display a demonstration grid board.* Write this information on the grid. Be certain that T is eight blocks east and six blocks north of H.



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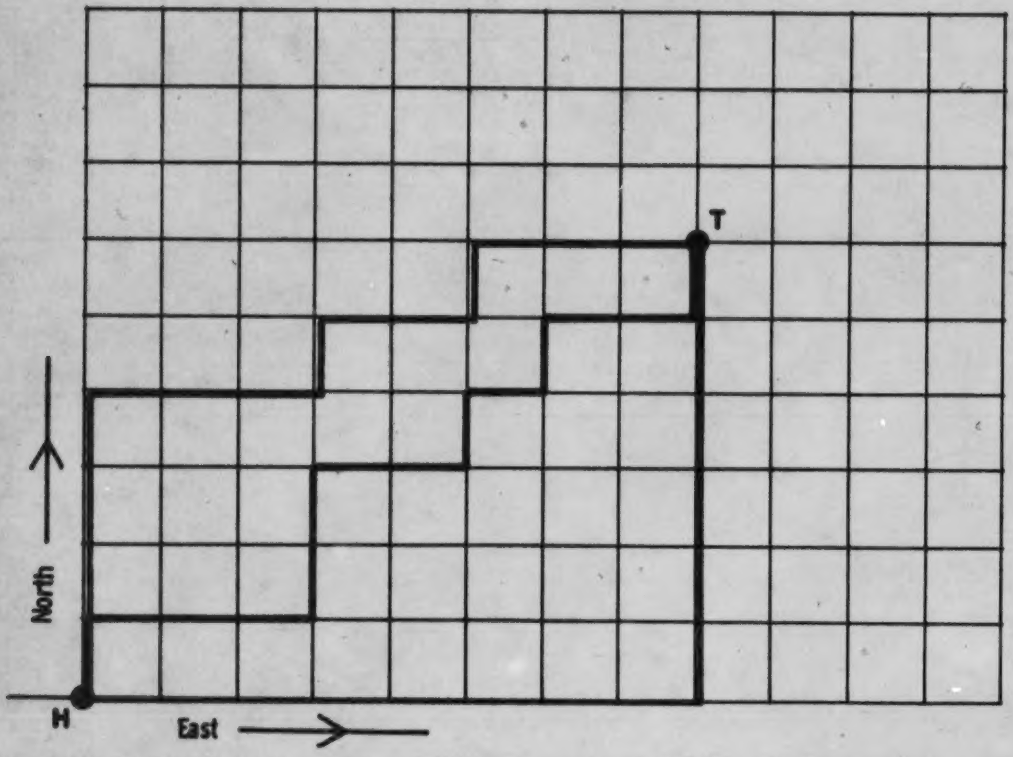
* The printed grid (Robot Walk Grid) provided in your classroom set of materials is ideal size for this lesson. Otherwise, use a demonstration grid board (see the NOTE ON GRIDS in the introduction to the Geometry strand).

W 9

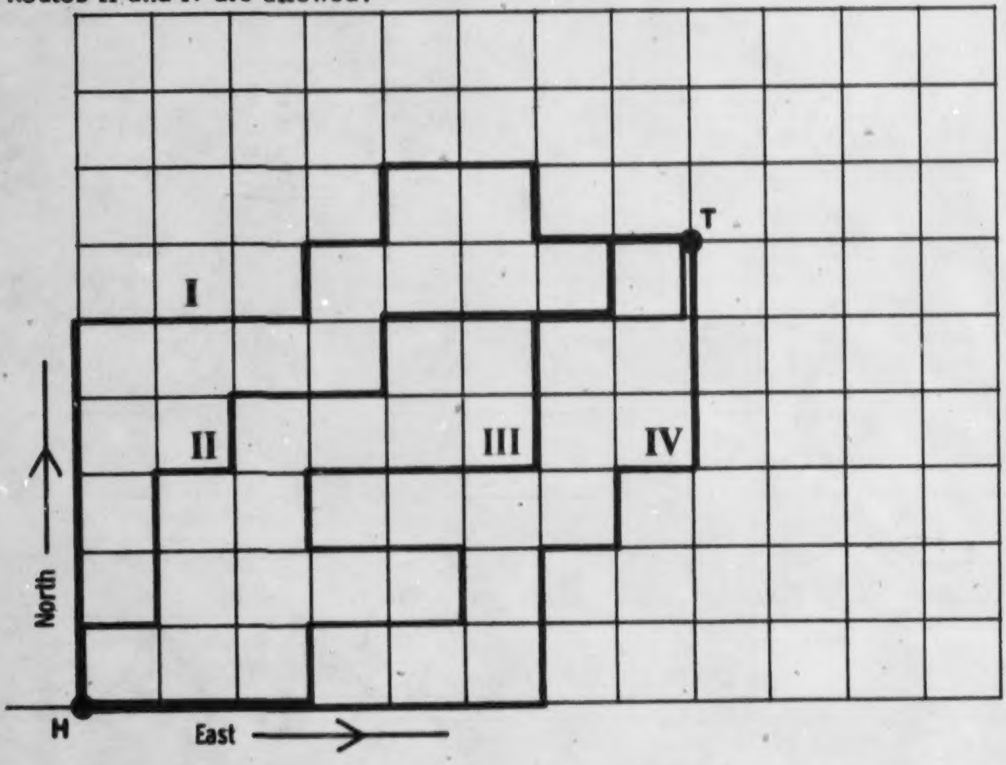
T: This grid is an enlargement of your worksheet. It is a street map of a little town. H is the house of a famous detective named "Spick". Some diamonds were stolen from Spick's house and Spick knows that this stolen treasure is hidden at T. In order to find clues about who the thieves are, Spick decides to explore all the routes from his house, H, to the hidden treasure, T. He has to be careful because all the streets are one-way in this part of the town. The roads only run north or east. Spick is driving, so he must stay on the streets. Who can trace some routes Spick could take from H to T?

Let several students trace routes that Spick could take from H to T following the rule that all roads are one-way, either to the north or to the east. Several examples of routes that Spick could take are shown in the next illustration.



Encourage the class to watch carefully for routes which break the one-way rule by going west or south along some streets; such rou[illegible] not allowed.

After this, trace several routes yourself on the poster and for each route ask the class if Spick could use that route. For example, you might trace in turn routes I, II, III, and IV as pictured below. Routes I and III are not allowed. Routes II and IV are allowed.



T: On one side of your worksheet, use colored pencils to draw at least three routes that Spick could take from H to T.

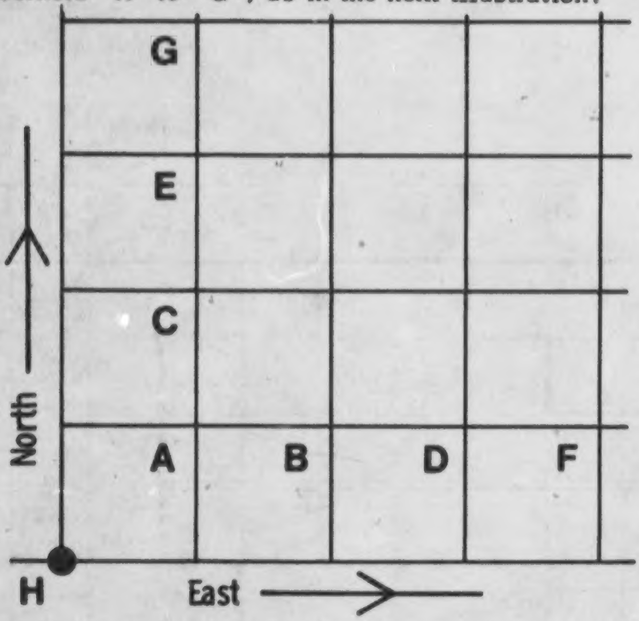
Observe the students' work as they draw these routes and individually point out routes which are not allowed.

T: How many different routes do you think Spick could take from H to T?

List some students' estimates on the board. Students in one of our development classes estimated 24, 72, 37, 80, and 48.

W 9

Draw an enlargement of the lower left part of the grid on the board and label appropriate corners "A" to "G", as in the next illustration.



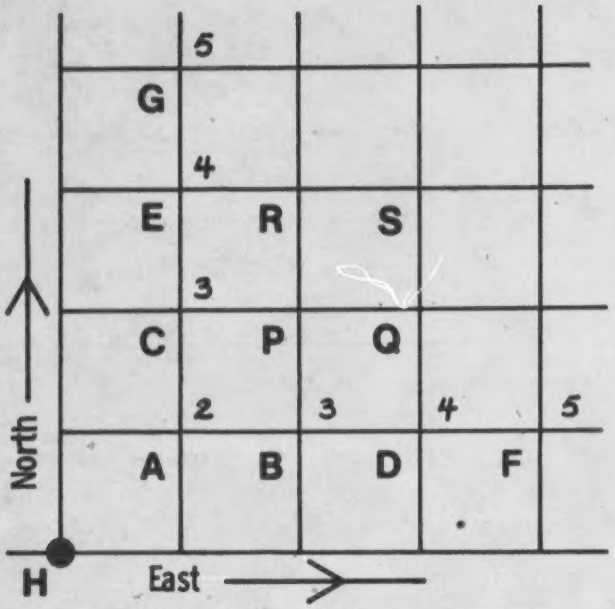
T (point to the "H" and "T" on the original map): This is an enlargement of part of the map. Our problem is to find the number of different routes Spick could take from H to T. First let's try to answer some easier questions using this enlarged part of the map. How many different routes are there from H to A? (Two)

Invite a student to trace the two routes from H to A, and then write "2" near the corner marked "A".

In a similar manner, ask the students to find the number of routes there are:

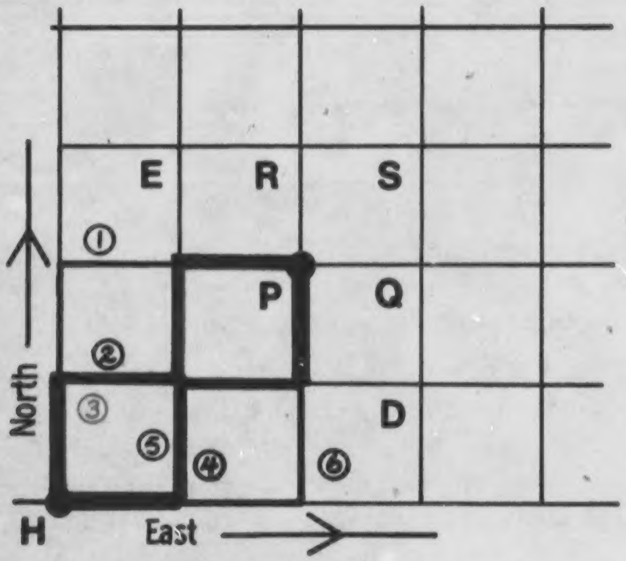
from H to B (three);
from H to C (three);
from H to D (four);
from H to E (four);
from H to F (five);
from H to G (five).

Record the answers on the grid and label appropriate corners "P", "Q", "R", and "S" as in the next illustration.


5
G
4
E
R
S
3
C
P
Q
2
3
4
5
A
B
D
F
North
H
East

T: How many routes are there from H to P? (Six) Who can trace them?

There may be disagreement so invite students to defend their answers by tracing all the routes from H to P. This is a fairly difficult task; encourage students to be systematic so as to find all six routes without duplication. The following illustration suggests one way to count the routes systematically.


E
R
S
①
P
Q
②
③
D
⑤
④
⑥
North
H
East

T: Can anyone convince me that there are six routes from H to P without having to trace the routes?

Perhaps some students will notice that all the routes from H to P must pass through B or C. Since there are three routes from H to P via B and three routes from H to P via C, there are in all six routes from H to P. If this explanation is not offered by the students, use the following dialogue to lead them to this observation.

T: How many roads from H to P pass through B?

S: Three; because there are three routes from H to B.

T: How many routes from H to P pass through C?

S: Three; because there are three routes from H to C?

T: Are there any routes from H to P which do not pass through either B or C?

S: No.

Write "6" near the corner labeled "P". Avoid further discussion of this situation. Instead, progress to the next corner.

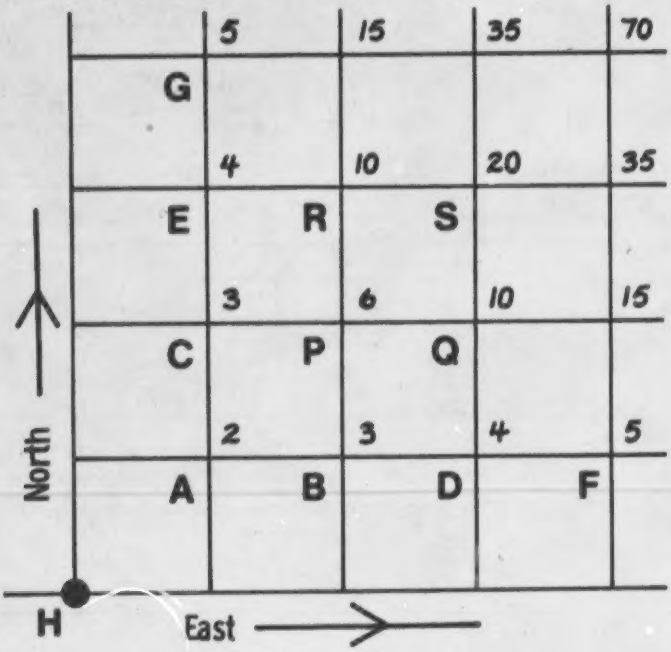
T: How many routes are there from H to Q?

S: Ten; because 4 + 6 = 10.

T: Why do you add 4 and 6?

S: All routes from H to Q must pass through either D or P. There are four routes from H to D and six routes from H to P. Therefore, there are ten routes from H to Q because 4 + 6 = 10.

In a similar manner, lead the class to discover that the number of routes from H to R is ten (10 = 6 + 4) and from H to S is twenty (20 = 10 + 10). Then ask about other corners until your students seem certain of the pattern. The solution for this enlarged part of the map is shown below.

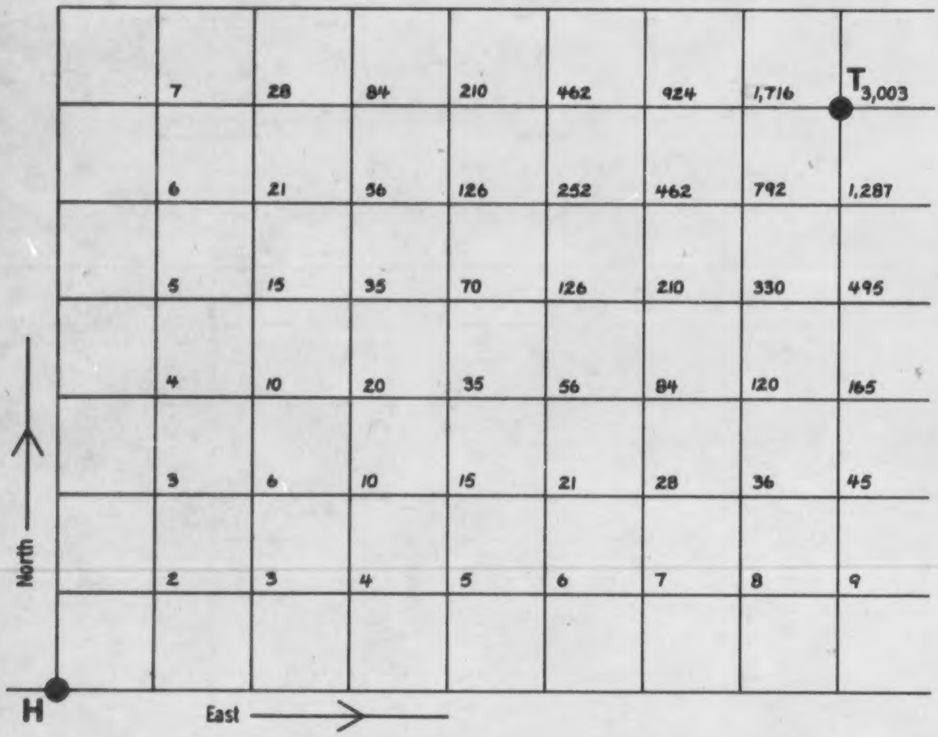


T: Use the pattern we discovered to find the number of routes from H to T. You can copy the numbers from this enlarged part of the map on the board onto your worksheet (reverse side). Then continue to fill in the numbers until you reach T.

As the class begins to work on this problem, copy the numbers from the enlarged part of the map on the board to the original map on your grid board. Check individual students' work and invite students to put correct numbers of routes on the demonstration map so as to gradually find the number of routes from H to each corner. The complete solution is shown in the next illustration. The students should find that there are 3,003 routes from H to T.

137

W 9



7	28	84	210	462	924	1,716	T 3,003
6	21	56	126	252	462	792	1,287
5	15	35	70	126	210	330	495
4	10	20	35	56	84	120	165
3	6	10	15	21	28	36	45
2	3	4	5	6	7	8	9

T: How many routes are there from H to T? (3,003)
Were your estimates of the answer very close?

Students are usually surprised at how large the number of routes from H to T is and how inaccurate they were in their estimates.

T: Our map is not complete. These corners (directly east of H) are not yet labeled. (Point to one of the corners directly east of H.) How many routes could Spick take to this corner?

S: Only one. He must drive directly east to it.

S: There is only one route to each of the corners directly east of H.

In a similar manner, conclude that there is only one route to each of the corners directly north of H. Record this information on the grid by writing "1" near each of these corners.

Distribute Worksheet N64 which gives the complete solution. Briefly, let the students check their own calculations.

T: Do you notice any patterns among the numbers on the map?

Allow students to describe any patterns they notice; it is likely they will make a variety of suggestions.

Perhaps some students will notice that most of the numbers appear twice on the grid. If no one makes this observation, use the following dialogue to lead the class to this discovery.

T: Can we pair some corners with the same number?

S: Yes, there are two corners with 28.

S: There are also two corners with 10, 56, 210, and many other numbers.

Once students observe many pairs of corners with the same number, extend the discussion of this idea. Point to the two corners labeled "10". Then point to the two corners labeled "84".

T: Why do certain pairs of corners have the same number of routes to each corner of the pair? Is there a pattern?

S: One corner labeled "10" is three blocks east of H and two blocks north of H. The other corner labeled "10" is two blocks east of H and three blocks north of H.

S: One corner labeled "84" is six blocks east and three blocks north of H. The other corner labeled "84" is just the opposite; it is three blocks east and six blocks north of H.

T: Which corners are not paired?

S: The corners labeled "2", "6", "20", "70", and so on.

T: Is the corner labeled "1,287" part of a pair?

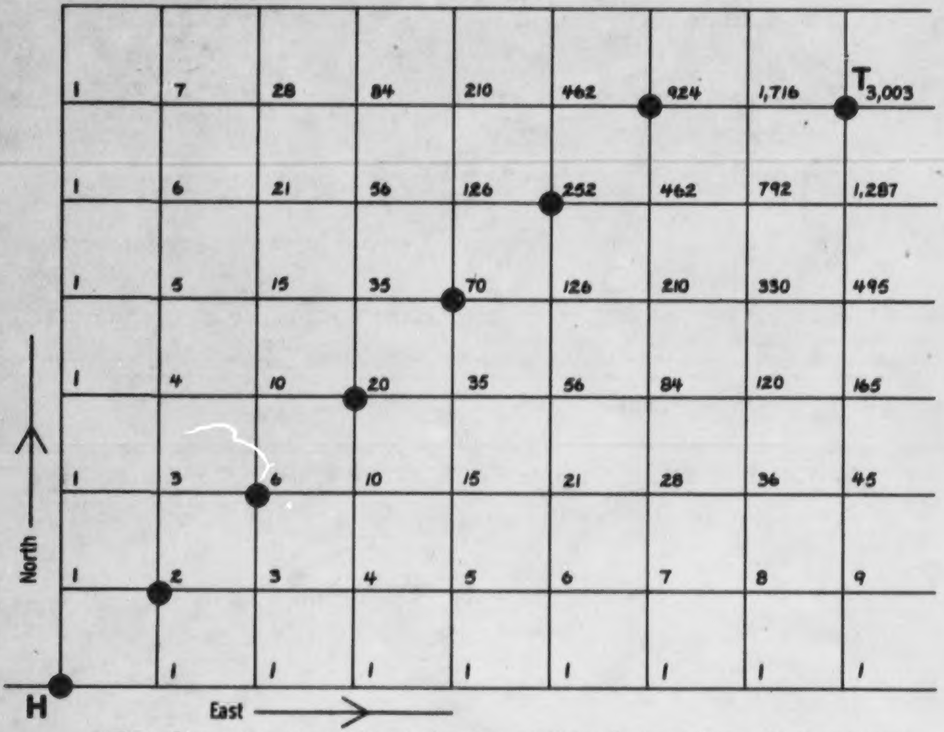
W 9

S: It would be part of a pair if the map were bigger.

T: Where would the other corner labeled "1,287" be?

S: Eight blocks north of H and five blocks east of H, because on the map the corner labeled "1,287" is five blocks north of H and eight blocks east of H.

Mark with red dots all the corners which are not part of any pair no matter how far you extend the map.



1	7	28	84	210	462	● 924	1,716	● T 3,003
1	6	21	56	126	● 252	462	792	1,287
1	5	15	35	● 70	126	210	330	495
1	4	10	● 20	35	56	84	120	165
1	3	● 6	10	15	21	28	36	45
1	● 2	3	4	5	6	7	8	9
● H	1	1	1	1	1	1	1	1

T: What do you notice about these red dots for numbers that are not paired?

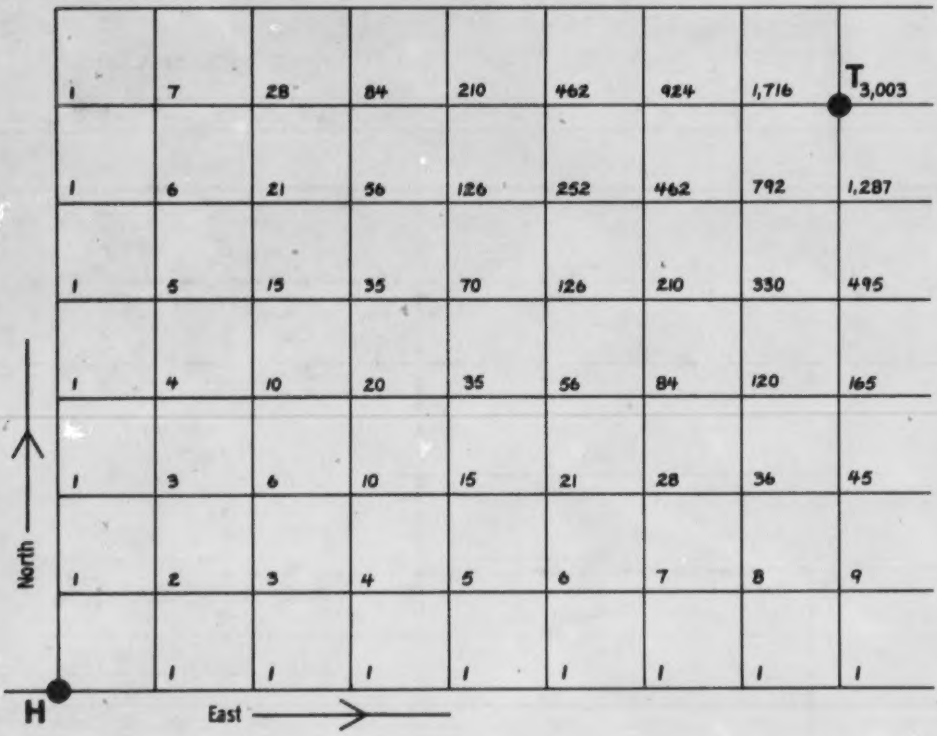
S: They are in a straight line.

S: The dots are on the corners that are: one block east and one block north of H; two blocks east and two blocks north of H; and so on.

If it is feasible, save the grid with the answers for use in Lesson 2. Collect Worksheet N64 for use also in Lesson 2.

Lesson 2

Distribute Worksheet N64 and display a grid labeled as in this illustration.



1	7	28	84	210	462	924	1,716	T 3,003
1	6	21	56	126	252	462	792	1,287
1	5	15	35	70	126	210	330	495
1	4	10	20	35	56	84	120	165
1	3	6	10	15	21	28	36	45
1	2	3	4	5	6	7	8	9
H	1	1	1	1	1	1	1	1

Ask a student to recall the story from last week about the detective Spick who decided to search all of the routes from his house, H, to the treasure, T. Review briefly how it was determined that there are 3,003 such routes along the one-way streets.

T: Spick traveled from his house at H to the treasure at T many times. He decided to write a secret code in his notebook for each route he took. One day Spick drove from H to T and wrote "**10110000111000**" in his notebook.

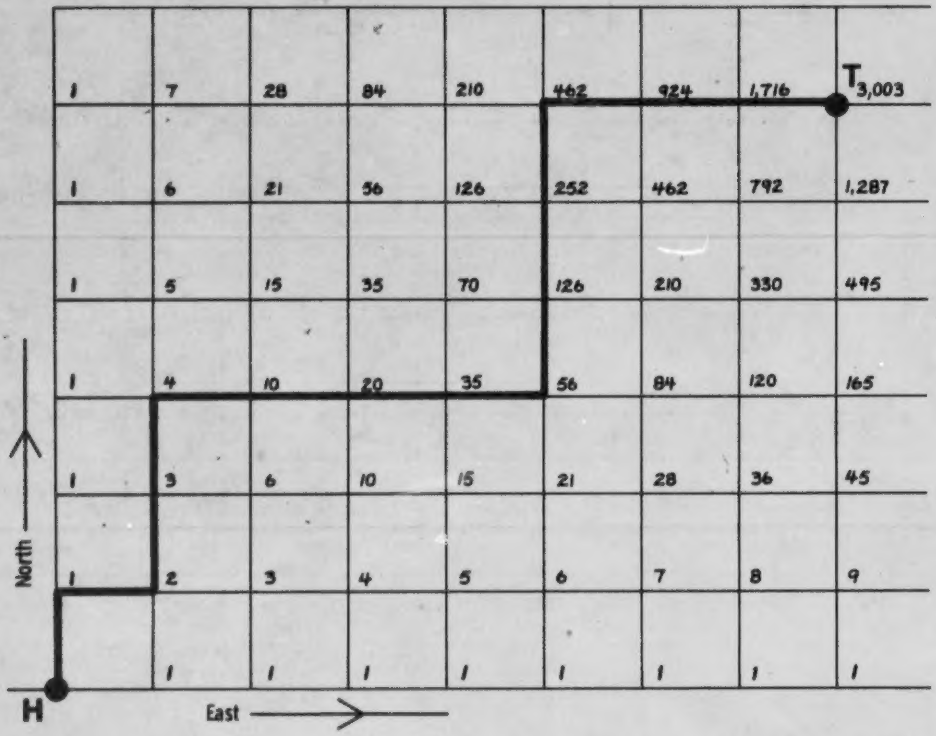
Write the code word on the board.

10110000111000

W 10

T: Can anyone guess Spick's secret rule for writing codewords? If you know the secret, keep it to yourself for the moment. Which route from H to T do you think Spick traveled when he wrote this codeword?

Let a student trace the route. The route below is correct because in the codeword each **1** means to go one block north and each **0** means to go one block east.



1 7 28 84 210 462 924 1,716 T 3,003
1 6 21 56 126 252 462 792 1,287
1 5 15 35 70 126 210 330 495
1 4 10 20 35 56 84 120 165
1 3 6 10 15 21 28 36 45
1 2 3 4 5 6 7 8 9
1 1 1 1 1 1 1 1
H
North
East

You may find it necessary to give the class some help towards finding this route. If necessary, read the first five numbers in the codeword, **10110**, as you trace the first five blocks of the route drawn in red in the above illustration. Trace the route from H to the corner labeled "10". Ask a student to complete the route.

T: Who can trace the whole route from H to T?

Very likely some students will be able to trace the route. If necessary, trace some more of the route.

Repeat this exercise by writing these codewords on the board and asking a student to trace the appropriate route for each codeword.

11000010101100

00010001010111

T: Who can write a codeword for another route from H to T?

A student might suggest 11000110010001.

T: Do you think that this codeword represents a route from H to T? Who can check it by tracing the route?

Let a student trace the route from H to T.

T: Who can explain the code?

S: Each 1 in the codeword means to go one block north. Each 0 means to go one block east.

T: Who can write a codeword for another route from H to T?

A student might suggest 10100000110101.

T: How can we check if this codeword represents a route from H to T?

S: Trace the route.

T: Can we check it without tracing the route?

S: Yes, each codeword must have 14 digits.

S: There must be exactly six 1 's and eight 0 's in each codeword.

T: Why?

S: Because T is six blocks north and eight blocks east of H.

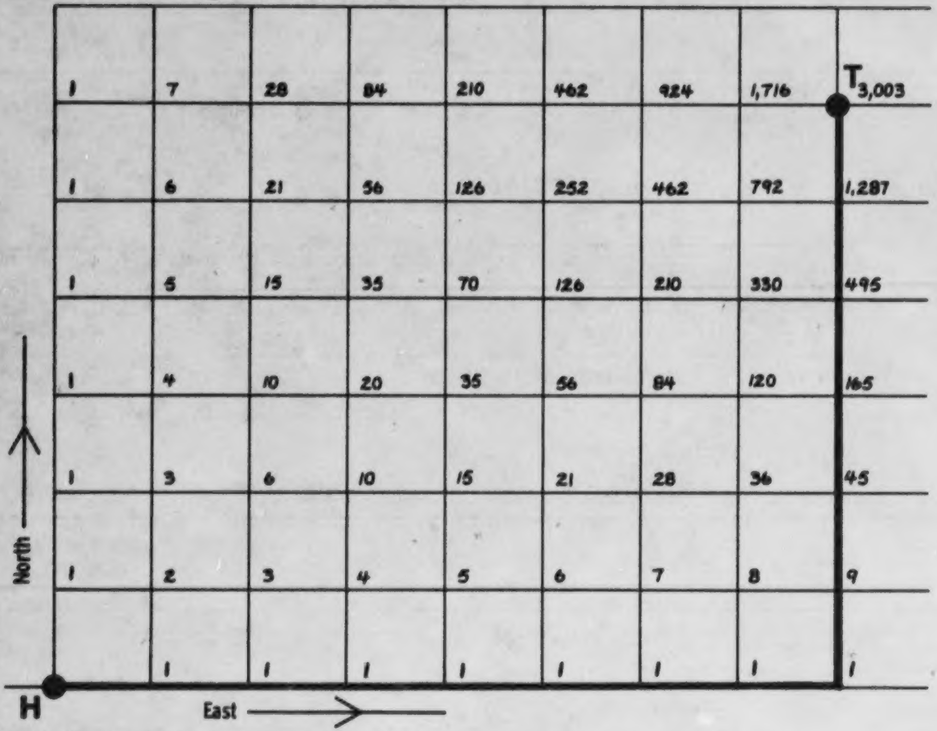
W 10

T: So is this codeword correct? (Yes) Who can trace the route?

Invite a student to trace the route.

T: I will trace a route. Write the codeword for this route as I trace it.

Trace the following route shown in red.


T
1 7 28 84 210 462 924 1,716 3,003
1 6 21 56 126 252 462 792 1,287
1 5 15 35 70 126 210 330 495
1 4 10 20 35 56 84 120 165
1 3 6 10 15 21 28 36 45
1 2 3 4 5 6 7 8 9
1 1 1 1 1 1 1 1
H
North
East

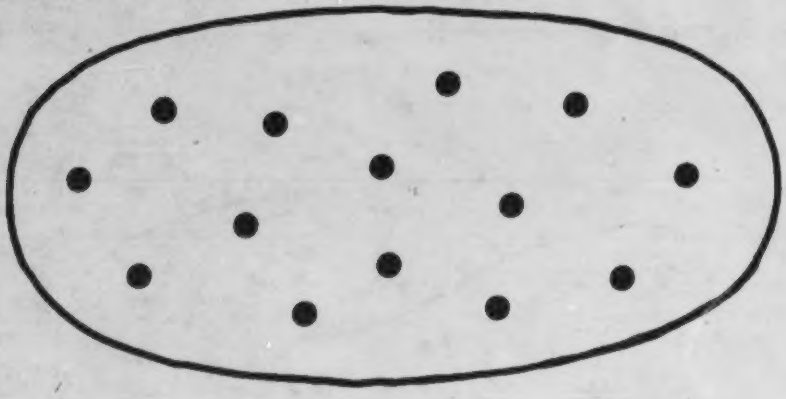
Check the answers of some students.

T: What is the codeword? **(00000000111111)**
How many codewords could we write with six 1's and eight 0's?

S: 3,003; because there is one codeword for each route from H to T and we know that there are 3,003 routes from H to T.

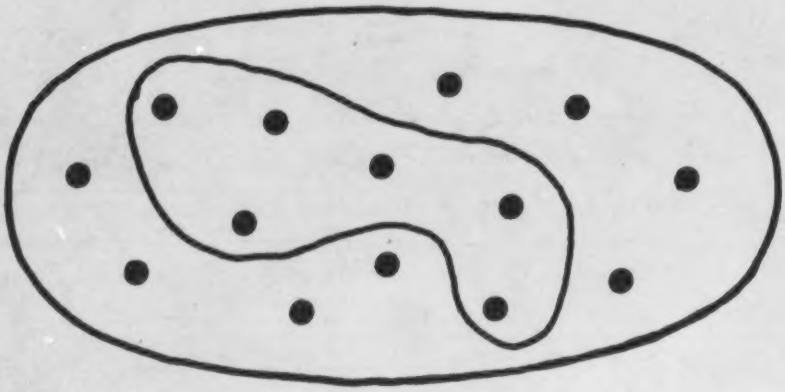
Briefly discuss this answer with your class. Your students should observe that each different codeword with six 1's and eight 0's describes a different route from H to T. Also, for each different route from H to T, there is one such codeword. Thus, there are 3,003 codewords with exactly six 1's and eight 0's.

Draw a string with fourteen dots in it on the board.



T: Spick also knew that exactly six thieves had stolen the treasure. Spick had fourteen suspects and was sure that all six thieves were among his suspects. He felt that they would confess if he could interview all six thieves together. So he decided to interview the fourteen suspects in groups of six.

Draw a red string around six of the dots in the string picture.



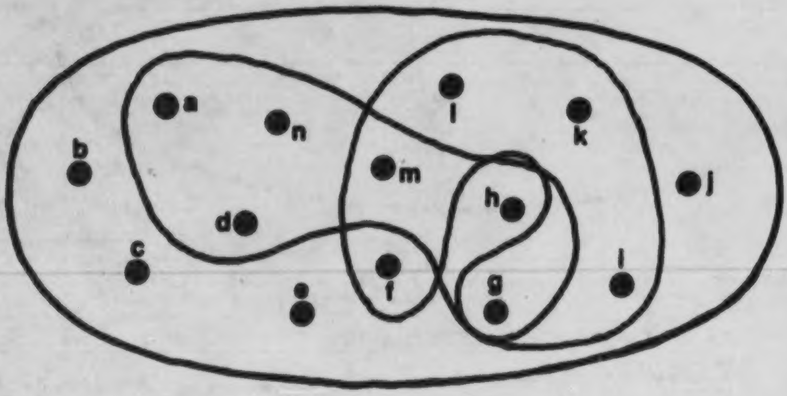
T: How many groups of six do you think Spick could interview?

Write your students' estimates on the board.

W 10

T: Spick realized that he would have to interview many groups of six suspects. He decided to write a secret codeword in his notebook for each group of suspects he interviewed.

Add the following information to the picture on the board.



a	b	c	d	e	f	g	h	i	j	k	l	m	n
1	0	0	1	0	0	1	1	0	0	0	0	1	1

T (point to the red digits): Spick wrote this codeword in his notebook when he interviewed the six suspects in the red string. Does anyone think they know Spick's secret rule for writing codewords? If you know his secret, keep it to yourself for the moment. Try to write the code- for the group of six suspects in the blue string.

Let a student write the codeword in blue for the blue string. The following codeword in blue is correct since the rule is to write a 1 for each suspect in the blue string and to write 0 for each suspect not in the blue string. Do not explain the rule yet.

a	b	c	d	e	f	g	h	i	j	k	l	m	n
1	0	0	1	0	0	1	1	0	0	0	0	1	1
0	0	0	0	0	1	1	0	1	0	1	1	1	0

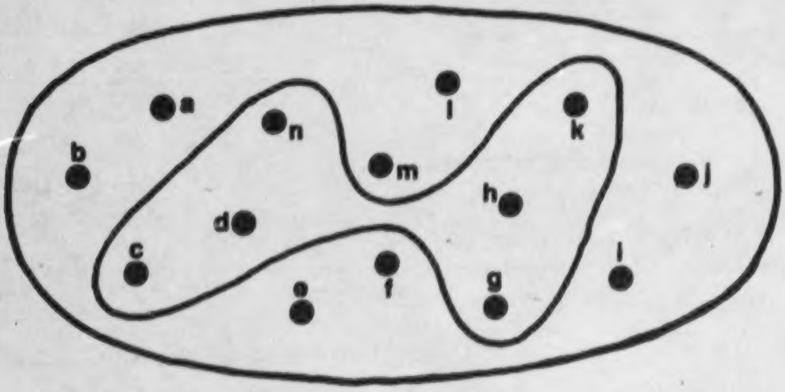
T: Who can write a codeword for another group of six suspects?

A student might suggest:

a	b	c	d	e	f	g	h	i	j	k	l	m	n
0	0	1	1	0	0	1	1	0	0	1	0	0	1

T: Is this a correct codeword for another group of six suspects? (Yes) Who can draw a string for this group?

Invite a student to draw the string for this group. The string should include exactly the six dots for the six suspects marked with a 1. The next illustration shows the correct string for our example.



T: Who can explain Spick's secret rule?

S: A 1 means that the suspect will be interviewed in this group of six. A 0 means that the suspect will not be interviewed in this group.

W 10

T: What do you notice about each of these codewords?

S: Each codeword has fourteen digits for the fourteen suspects. There must be six 1's for the suspects to be interviewed and eight 0's for the other suspects.

Erase all the inside strings from the string picture.

Invite a student to write another codeword for a group of six suspects and then ask another student to draw the corresponding string. Then reverse the problem by inviting one student to draw a string for six suspects and then asking another student to write the corresponding codeword.

T: For each string for a group of six suspects, there is a codeword with six 1's and eight 0's. For each codeword of this type, you can draw the corresponding string. How many groups of six are there among the fourteen suspects?

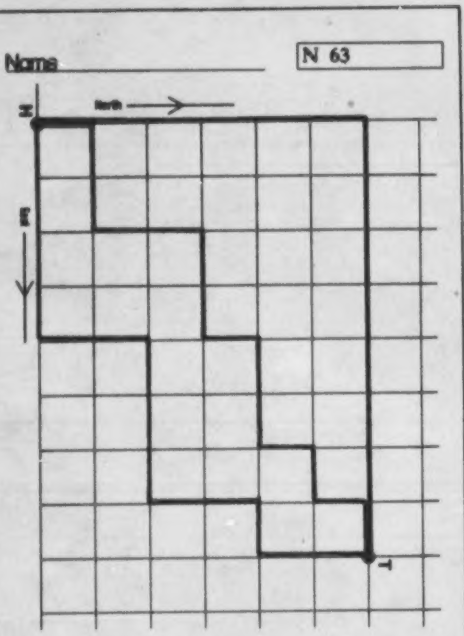
S: 3,003; since we know from the previous problem that there are 3,003 codewords with exactly six 1's and eight 0's.

S: Each road from H to T on Spick's map corresponds to a codeword with six 1's and eight 0's. Each string for six suspects corresponds to a similar codeword with six 1's and eight 0's. Since we figured out that there are 3,003 roads from H to T, there are 3,003 groups of six suspects.

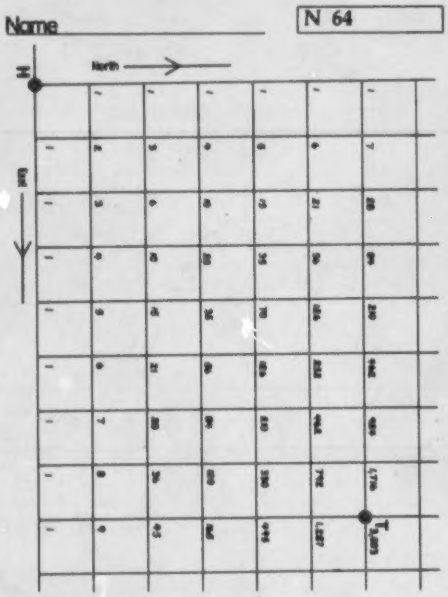
S: The two problems are very similar.

Compare the answer, 3,003, to your students' estimates.

Distribute the storybook "The Hidden Treasure". In the time remaining, read and discuss the story. Encourage students to read the storybook on their own if there is not time to finish it in class. The story reviews the two lessons about Spick, except that in the story the map of Spick's town is smaller (T is only six blocks east and four blocks north of H) and there are six thieves among only ten suspects. Thus in the storybook there are 210 routes from H to T and there are 210 groups of six thieves among ten suspects.


Name
N 63
H
North
East
T

(There are many paths (3,003 in all) from H to T.)


Name
N 64
H
North
East
T

A
SELECTION
OF
PROBLEMS

#4

A SELECTION OF PROBLEMS #4

The problems in this workbook are in order of increasing difficulty. Approximately the first ten pages of the workbook have easy problems, the next ten to twelve pages have average problems, and the last ten pages have more challenging problems. It is suggested that all students work through as many pages as they can handle during the two lessons scheduled for the workbook. Not all students are expected to complete the entire workbook; some will succeed to do only the easy problems, many should finish about twenty pages, and only the faster students will reach the most challenging problems at the end of the booklet.

Many problems in the workbook were designed with the availability of hand-calculators in mind. If possible, provide all the students with hand-calculators or at least give students access to hand-calculators.* You will find that many students do not need or want to use calculators; however, some students still need the support of the calculator in thinking about these problems.

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* See Section 2(f) of the General Introduction for additional comments on the use of hand-calculators and on desirable characteristics of hand-calculators.

W 11 & 12

Lesson 1

Give each student a copy of the workbook "A Selection of Problems #4" and let them work on their own.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the end of this lesson, collect the workbooks and have them ready to return for Lesson 2.

Lesson 2

If many students had difficulty with the subtraction algorithm puzzles on page 7 of "A Selection of Problems #4", present the following short exercise. Otherwise, return the workbooks to the students, ask them to correct their mistakes and to continue working.

Write these three problems on the board and ask the students to copy and solve them.

$$\begin{array}{r} 4{,}5\ 8\ 2 \\ -\quad 7\ 5\ 9 \\ \hline \end{array} \qquad \begin{array}{r} \square\ 4\ 8\ 3 \\ -1\ 7\ \square\ \square \\ \hline 4\ \square\ 5\ 9 \end{array} \qquad \begin{array}{r} \square\ 2\ \square \\ -\quad \square\ 5 \\ \hline 5\ 7\ 8 \end{array}$$

T: Remember that here you may only put one of the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in a box.

When most of the class are finished, invite students to present their solutions at the board.

Solutions are given below.

$$\begin{array}{r} 4,5\;8\;2 \\ -\quad 7\;5\;9 \\ \hline 3,8\;2\;3 \end{array} \qquad \begin{array}{r} \boxed{6}\;4\;8\;3 \\ -1\;\;7\,\boxed{2}\,\boxed{4} \\ \hline 4\,\boxed{7}\;5\;9 \end{array} \qquad \begin{array}{r} \boxed{6}\;2\,\boxed{3} \\ -\quad \boxed{4}\;5 \\ \hline 5\;7\;8 \end{array}$$

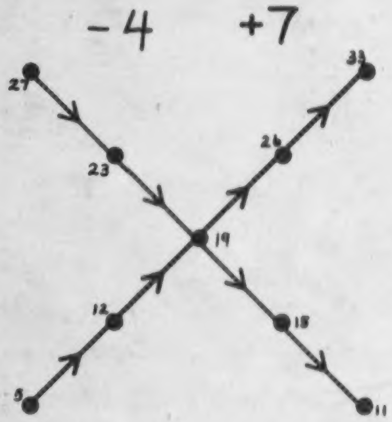
Return the students' copies of "A Selection of Problems #4". You may wish to discuss any pages that were difficult for most of the students. Ask the students to correct their mistakes, if any, and then to continue working in their workbooks.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion of that problem.

At the end of this lesson, collect the workbooks for your progress check. You may wish to let some students work further in their workbooks later during any free study time.

A SELECTION OF PROBLEMS #4

Label the dots.


-4
+7
27
23
19
15
11
33
26
12
5

Which is the largest even number in this picture? 26

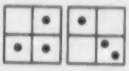
Which is the smallest multiple of 3 in this picture? 12

2

Pev is a secret number.

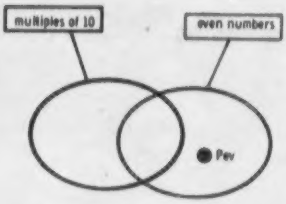
First clue

Pev can be shown on these Minicomputer boards by removing exactly one checker.



Pev could be 40, 60, 70, 72, or 79.

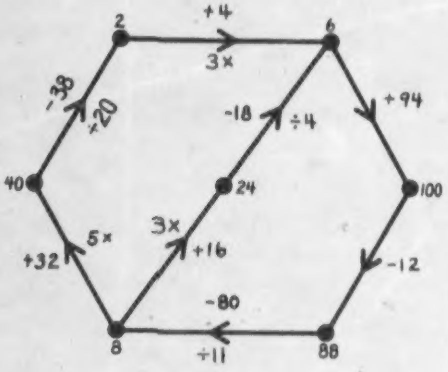
Second clue


multiples of 10
even numbers
Pev

Who is Pev? 72

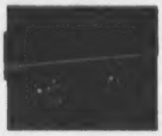
3

Label each red arrow. Try to label some arrows in two ways. One is done for you. Many answers are possible.


2
+4
3×
6
+94
-38
÷20
-18
÷4
40
24
100
5×
+32
3×
+16
-12
-80
8
÷11
88

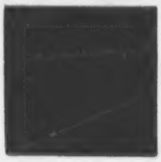
4

(Many solutions are possible.)



How many squares of this size, ■, could fit into the red region? 13

How many squares of this size, ■, could fit into the blue region? 17

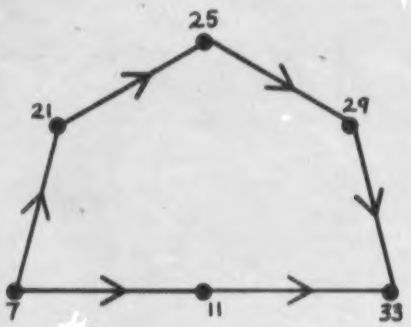


How many triangles of this size, ◢, could fit into the red region? 12

How many triangles of this size, ◢, could fit into the blue region? 6

5

Label the dots and fill in the box for the red arrows.



+ $\boxed{4}$ 3×

Put one of the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in each box to make the calculations correct. You may use a digit more than once.

Add

$$\begin{array}{r} \boxed{5}\,8\,2\,5 \\ +\,6\,3\,\boxed{5}\,7 \\ \hline 1\,2\,\boxed{1}\,8\,\boxed{2} \end{array}$$

Add

$$\begin{array}{r} 6\,\boxed{4}\,6\,9 \\ +\,3\,\boxed{8}\,8\,7\,\boxed{7} \\ \hline \boxed{4}\,4\,3\,\boxed{3}\,2 \end{array}$$

Add

$$\begin{array}{r} 8\,\boxed{3}\,3\,6 \\ \boxed{4}\,9 \\ +\,\boxed{3}\,\boxed{7}\,2\,5\,7 \\ \hline 4\,5\,6\,4\,\boxed{2} \end{array}$$

Subtract

$$\begin{array}{r} 4\,\boxed{0}\,\boxed{8}\,5\,8 \\ -\quad 3\,\boxed{3}\,2 \\ \hline \boxed{4}\,0\,5\,2\,\boxed{6} \end{array}$$

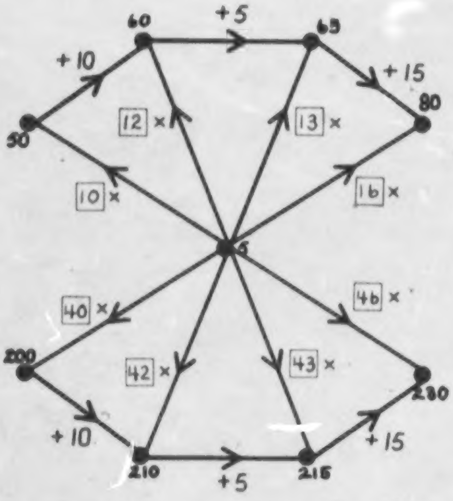
Subtract

$$\begin{array}{r} \boxed{9}\,6\,\boxed{1}\,7\,4 \\ -\,2\,\boxed{5}\,8\,\boxed{2}\,8 \\ \hline 7\,0\,3\,4\,\boxed{6} \end{array}$$

Subtract

$$\begin{array}{r} \boxed{6}\,5\,2 \\ -\,2\,8\,\boxed{3} \\ \hline 3\,\boxed{6}\,9 \end{array}$$

Label the dots and fill in the boxes for the blue arrows.



Put each number on the display of a hand-calculator using just these keys:

$\boxed{5}$, $\boxed{6}$, $\boxed{8}$, $\boxed{9}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, $\boxed{=}$.

Write the keys in the order you use them. You may use a key more than once.

It costs 1¢ each time you press a key. Try to spend less than the amount shown for each number.

$\boxed{5}\boxed{+}\boxed{9}\boxed{5}\boxed{=}$ 100 [8¢]

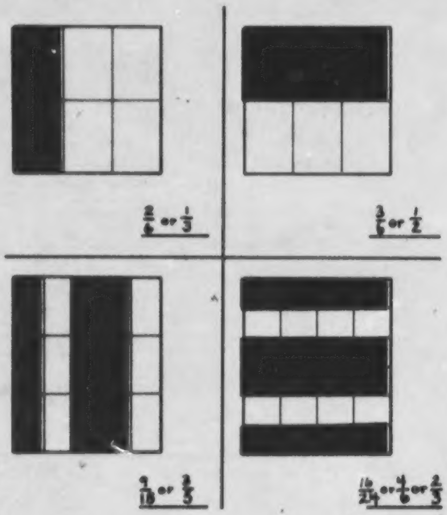
$\boxed{5}\boxed{+}\boxed{5}\boxed{\times}\boxed{=}\boxed{=}$ 1,000 [13¢]

$\boxed{9}\boxed{9}\boxed{9}\boxed{-}\boxed{6}\boxed{6}\boxed{6}\boxed{=}$ 333 [10¢]

$\boxed{9}\boxed{+}\boxed{9}\boxed{-}\boxed{8}\boxed{=}$ −7 [10¢]

(Many solutions are possible. See Section 2(f) of the General Introduction for comments on how calculator characteristics may effect the solutions.)

Write a fraction for each picture to indicate what part of the rectangle is shaded red.

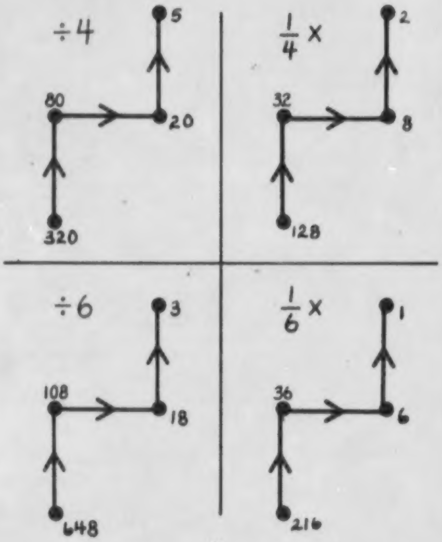


$\frac{2}{6}$ or $\frac{1}{3}$
$\frac{3}{6}$ or $\frac{1}{2}$
$\frac{9}{15}$ or $\frac{3}{5}$
$\frac{16}{24}$ or $\frac{4}{6}$ or $\frac{2}{3}$

10

(The fractions given here are suggested by the pictures; however, other correct names could be given.)

Label the dots.



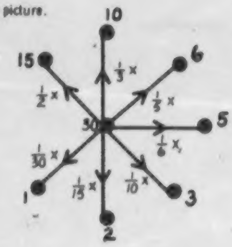
÷4
5
80
20
320
$\frac{1}{4}$ ×
2
32
8
128
÷6
3
108
18
648
$\frac{1}{6}$ ×
1
36
6
216

11

Bom is a secret number.

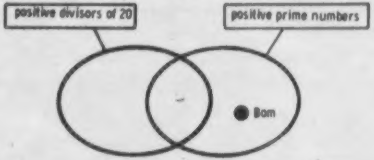
First clue

Bom is in this arrow picture.



10
15
6
$\frac{1}{3}$ ×
$\frac{1}{2}$ ×
$\frac{1}{5}$ ×
30
5
$\frac{1}{6}$ ×
$\frac{1}{30}$ ×
$\frac{1}{15}$ ×
$\frac{1}{10}$ ×
1
3
2

Second clue

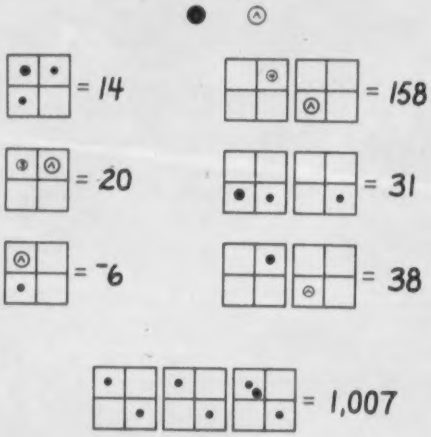


positive divisors of 20
positive prime numbers
Bom

Who is Bom? 3

12

Put each number on the Minicomputer by adding exactly one of these checkers.

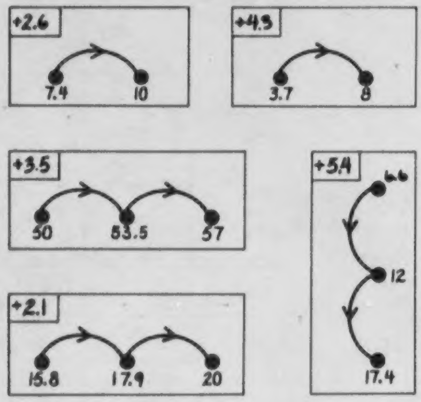


= 14
= 158
= 20
= 31
= ⁻6
= 38
= 1,007

13

For each arrow picture, fill in the box with the correct tag. Then label the dots.

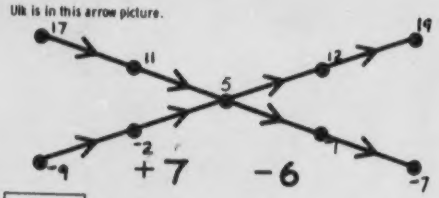
+3.5 +2.6 +5.4 +4.3 +2.1


+2.6
7.4
10
+4.3
3.7
8
+3.5
50
53.5
57
+5.4
6.6
12
17.4
+2.1
15.8
17.9
20

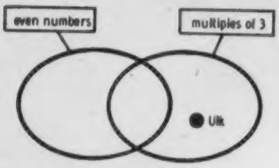
Ulk is a secret number.

First clue

Ulk is in this arrow picture.

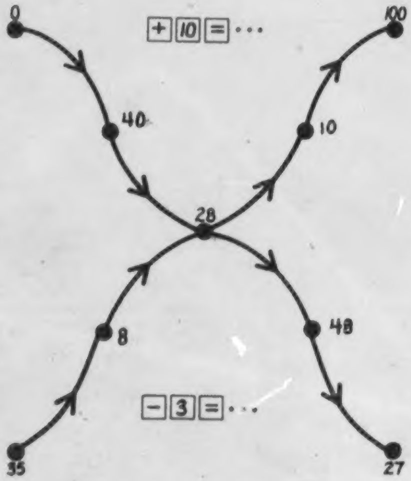

17
11
5
⁻1
⁻7
⁻9
⁻2
12
19
+7
−6

Second clue


even numbers
multiples of 3
Ulk

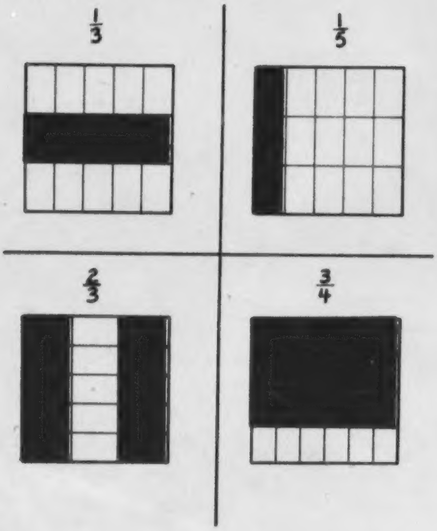
Ulk is ⁻9.

Label the dots. Many answers are possible.


0
40
28
48
27
35
8
10
100
+ 10 = ...
− 3 = ...

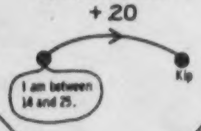
(Many solutions are possible.)

Color in red the indicated part of each rectangle.

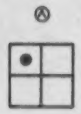

$\frac{1}{3}$
$\frac{1}{5}$
$\frac{2}{3}$
$\frac{3}{4}$

(Many solutions are possible.)

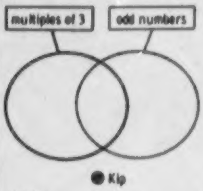
Kip is a whole number.


+ 20
I am between 14 and 25.
Kip

Kip can be put on this ones' board of the Minicomputer by adding exactly one negative checker.



Kip is in this string picture.

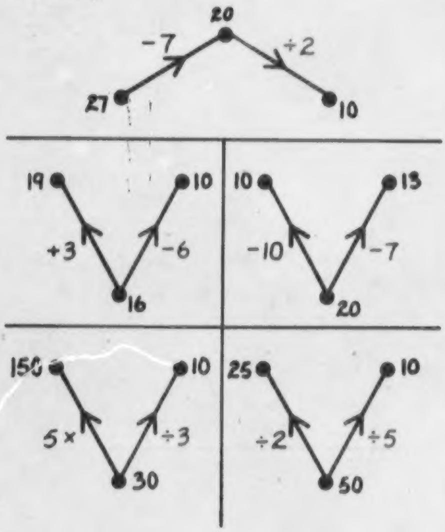

multiples of 3
odd numbers
Kip

Who is Kip? 38

18

---

10 is the smallest number in each arrow picture. Find 10 in each picture. Label all the dots.

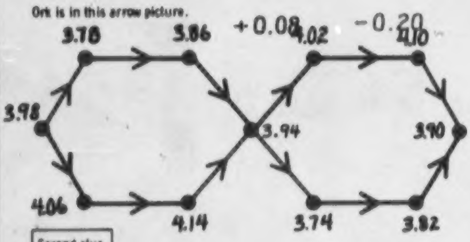

20
−7
÷2
27
10
19
10
+3
−6
16
10
15
−10
−7
20
150
10
5 ×
÷3
30
25
10
÷2
÷5
50

19

Ork is a secret number.

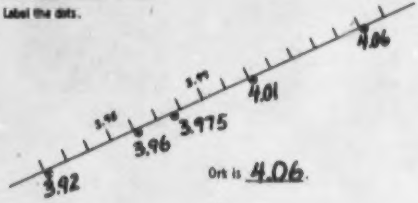
First clue

Ork is in this arrow picture.


3.78
3.86
+0.08
4.02
−0.20
4.10
3.98
3.94
3.90
4.06
4.14
3.74
3.82

Second clue

Ork is one of these red dots.
Label the dots.


4.06
3.99
4.01
3.98
3.975
3.96
3.92

Ork is 4.06.

20

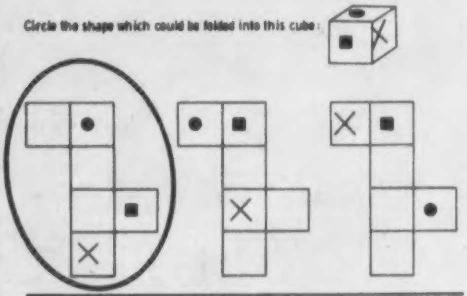
Put one of these symbols: + , − , × , ÷ in each blank box to make the hand-calculator sentence true. A symbol may be used twice in one sentence.

8	×	8	÷	8	=	8
18	÷	6	−	5	=	⁻2
6	−	0.8	+	10	=	15.2
12	×	0.5	×	2	=	12

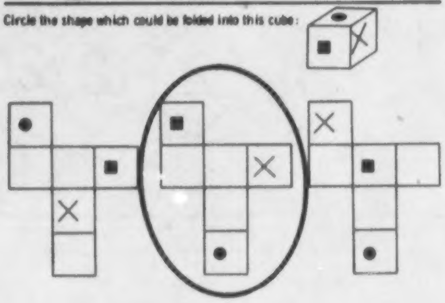
21

(Other solutions are possible.)

Circle the shape which could be folded into this cube:



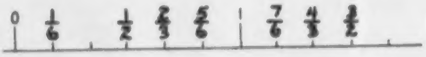
Circle the shape which could be folded into this cube:



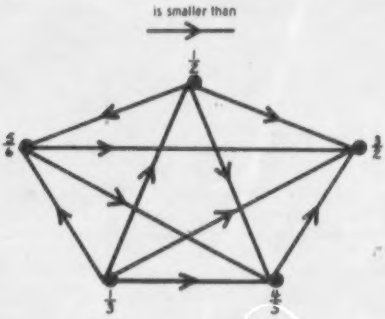
22

Locate these numbers on the number line.

$\frac{1}{2}$ $\frac{1}{6}$ $\frac{5}{6}$ $\frac{3}{2}$ $\frac{2}{3}$ $\frac{7}{6}$ $\frac{4}{3}$


0
1/6
1/2
2/3
5/6
1
7/6
4/3
3/2

Draw all the missing red arrows.

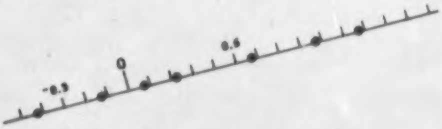

is smaller than
1/2
5/6
3/2
1/3
4/5

23

Zeb is a secret number.

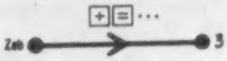
First clue

Zeb is one of these red dots.



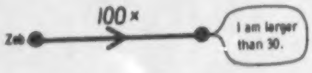
Zeb could be ⁻0.4, ⁻0.1, 0.1, 0.25, 0.6, 0.9, or 1.1.

Second clue



Zeb could be 0.1, 0.25, or 0.6.

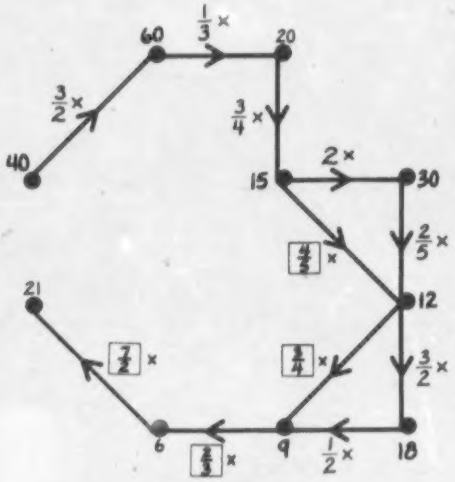
Third clue


100 ×
Zeb
I am larger than 30.

Who is Zeb? 0.6

24

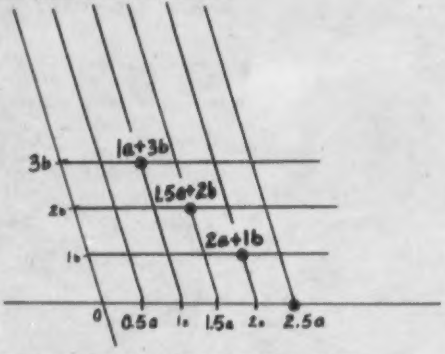
Label the dots and fill in the boxes for the red arrows.


40
3/2 ×
60
1/3 ×
20
3/4 ×
15
2 ×
30
4/5 ×
2/5 ×
12
3/2 ×
3/4 ×
18
1/2 ×
9
2/3 ×
6
7/2 ×
21

25

(Other correct labels are possible for the arrows; e.g., $\frac{4}{5} = \frac{8}{10} = \frac{12}{15} = \ldots.$)

One kilogram of apples (1a) costs 40¢.
One kilogram of bananas (1b) costs 20¢.


1a+3b
1.5a+2b
2a+1b
3b
2b
1b
0
0.5a
1a
1.5a
2a
2.5a

You must spend exactly $1.00. List some possible purchases of apples and bananas you could make. One is done for you.

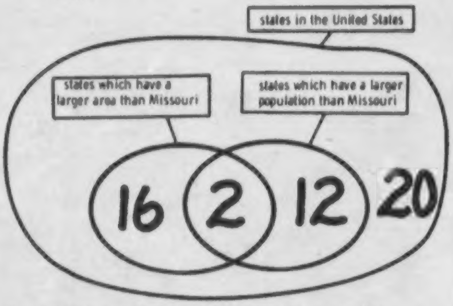
2a + 1b ; 1a + 3b ; 2.5a ; 1.5a + 2b .
(Many solutions are possible.)

Draw a dot on the picture for each of these purchases.

What do you notice about the dots? The four dots for a $1.00 purchase are on a line.

There are 50 states in the United States. Missouri is one of them. None of the other 49 states have exactly the same area or the same population as Missouri.

14 states have a larger population than Missouri (in 1975).
18 states have a larger area than Missouri.
20 states have both a smaller area and a smaller population than Missouri.

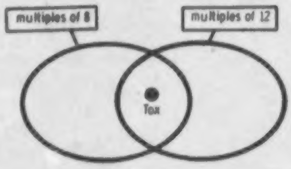

states in the United States
states which have a larger area than Missouri
states which have a larger population than Missouri
16
2
12
20

How many states have both a larger population and a larger area than Missouri? 2

How many states have both a larger area and a smaller population than Missouri? 16

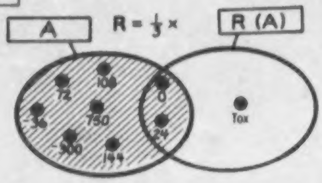
Tox is a secret number.

First clue


multiples of 8
multiples of 12
Tox

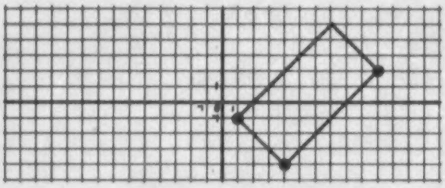
Tox could be 0 , 24 , $^-24$, 48 , $^-48$, 72 , and so on.

Second clue

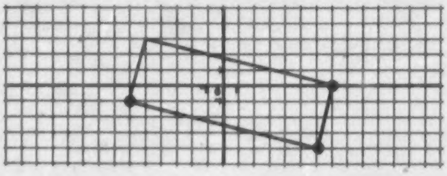

A
R = $\frac{1}{3}$ ×
R (A)
72
108
0
$^-36$
750
24
$^-300$
144
Tox

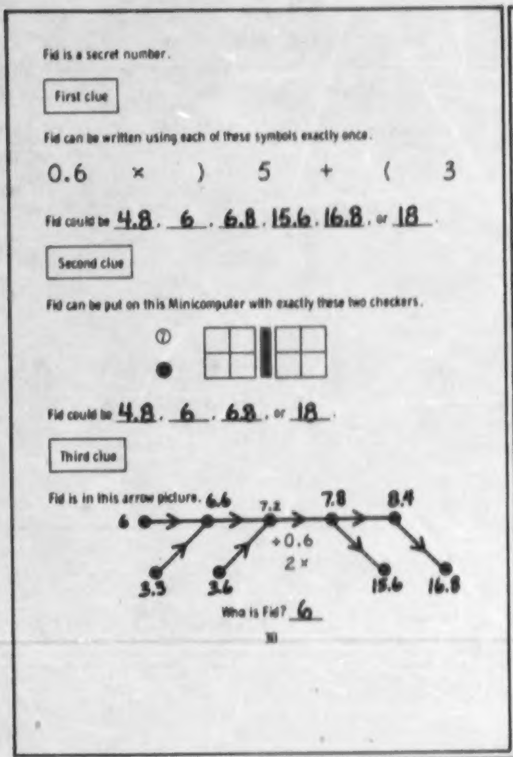
Who is Tox? 48

Three corners of a rectangle are at (10, 2), (1, $^-1$), and (4, $^-4$).
Draw dots at these three corners and then draw the rectangle.
Where is the fourth corner? (7,5)



Three corners of a rectangle are at (7, 0), (6, $^-4$), and ($^-6$, $^-1$).
Draw dots at these three corners and then draw the rectangle.
Where is the fourth corner? ($^-5$,3)





Fid is a secret number.

First clue

Fid can be written using each of these symbols exactly once.

0.6 ×) 5 + (3

Fid could be 4.8, 6, 6.8, 15.6, 16.8, or 18.

Second clue

Fid can be put on this Minicomputer with exactly these two checkers.

Fid could be 4.8, 6, 6.8, or 18.

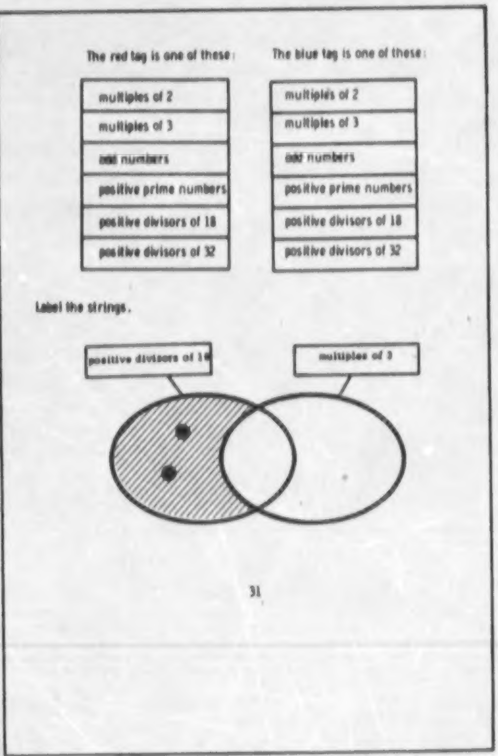
Third clue

Fid is in this arrow picture.

⑦
6
6.6
7.2
7.8
8.4
+0.6
2×
3.3
3.6
15.6
16.8

Who is Fid? 6

30

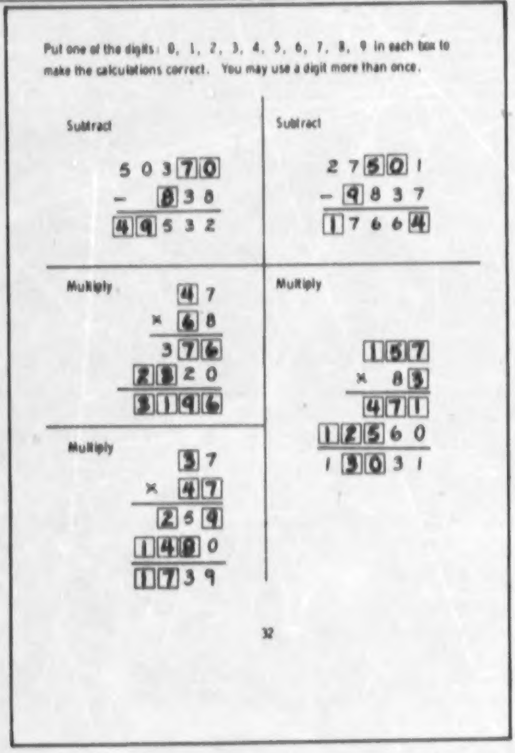


The red tag is one of these:	The blue tag is one of these:
multiples of 2	multiples of 2
multiples of 3	multiples of 3
odd numbers	odd numbers
positive prime numbers	positive prime numbers
positive divisors of 18	positive divisors of 18
positive divisors of 32	positive divisors of 32

Label the strings.

positive divisors of 18
multiples of 3

31



Put one of the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in each box to make the calculations correct. You may use a digit more than once.

Subtract

$$\begin{array}{r} 5\,0\,3\,\boxed{7}\,\boxed{0} \\ -\ \ \boxed{8}\,3\,8 \\ \hline \boxed{4}\,\boxed{9}\,5\,3\,2 \end{array}$$

Subtract

$$\begin{array}{r} 2\,7\,\boxed{5}\,\boxed{0}\,1 \\ -\ \boxed{9}\,8\,3\,7 \\ \hline \boxed{1}\,7\,6\,6\,\boxed{4} \end{array}$$

Multiply

$$\begin{array}{r} \boxed{4}\,7 \\ \times\ \boxed{6}\,8 \\ \hline 3\,\boxed{7}\,\boxed{6} \\ \boxed{2}\,\boxed{8}\,2\,0 \\ \hline \boxed{3}\,\boxed{1}\,\boxed{9}\,\boxed{6} \end{array}$$

Multiply

$$\begin{array}{r} \boxed{1}\,\boxed{5}\,\boxed{7} \\ \times\ \ 8\,\boxed{3} \\ \hline \boxed{4}\,\boxed{7}\,\boxed{1} \\ \boxed{1}\,\boxed{2}\,\boxed{5}\,6\,0 \\ \hline 1\,\boxed{3}\,\boxed{0}\,3\,1 \end{array}$$

Multiply

$$\begin{array}{r} \boxed{3}\,7 \\ \times\ \boxed{4}\,\boxed{7} \\ \hline \boxed{2}\,5\,\boxed{9} \\ \boxed{1}\,\boxed{4}\,\boxed{8}\,0 \\ \hline \boxed{1}\,\boxed{7}\,3\,9 \end{array}$$

32

A SELECTION OF PROBLEMS

#5

A SELECTION OF PROBLEMS #5

The problems in this workbook are in order of increasing difficulty. Approximately the first ten pages of the workbook have easy problems, the next ten to twelve pages have average problems, and the last ten pages have more challenging problems. It is suggested that all students work through as many pages as they can handle during the two lessons scheduled for the workbook. Not all students are expected to complete the entire workbook; some will succeed to do only the easy problems, many should finish about twenty pages, and only the faster students will reach the most challenging problems at the end of the booklet.

Many problems in the workbook were designed with the availability of hand-calculators in mind. If possible, provide all the students with hand-calculators or at least give students access to hand-calculators.[#] You will find that many students do not need or want to use calculators; however, some students still need the support of the calculator in thinking about these problems.

[#] See Section 2(f) of the General Introduction for additional comments on the use of hand-calculators and on desirable characteristics of hand-calculators.

W 13 & 14

Lesson 1

Give each student a copy of the workbook "A Selection of Problems #5" and let them work on their own.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the end of this lesson, collect the workbooks and have them ready to return for Lesson 2.

Lesson 2

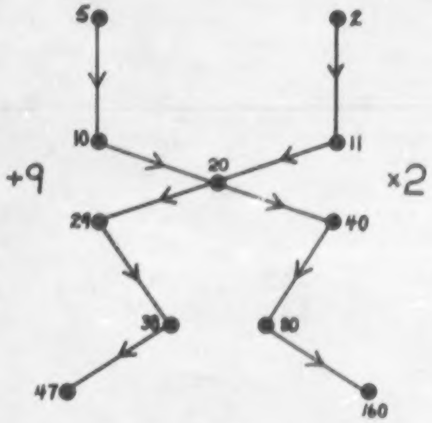
Return the students' copies of "A Selection of Problems #5". You may wish to discuss any page that was difficult for most of the students. Then ask the students to correct their mistakes, if any, and to continue working in their workbooks.

If you observe many students struggling with the same problem (or part of a problem), you could have a short discussion of that problem.

At the end of this lesson, collect the workbooks for your progress check. You may wish to let some students work further in their workbooks later during any free study time.

A SELECTION OF PROBLEMS #5

Label the dots.



Which is the largest odd number in this picture? 47

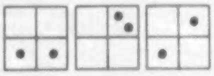
Which is the largest multiple of 5 in this picture? 160

2

Fum is a secret number.

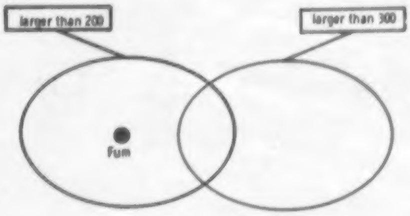
First clue

Fum can be shown on these Minicomputer boards by removing exactly one checker.



Fum could be 186, 286, 346, 382, or 384.

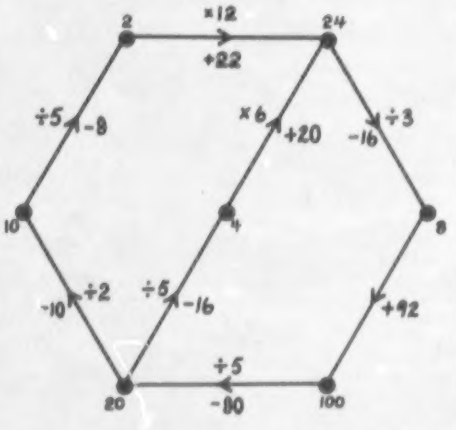
Second clue



Fum is 286.

3

Label the red arrows. Try to label some arrows in two ways. One is done for you. Many answers are possible.



4

(Other solutions are possible.)

Put one of the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9 in each box to make the calculations correct. You may use a digit as often as you wish.

Add

$$\begin{array}{r} 849\boxed{4} \\ +\ \boxed{2}53 \\ \hline \boxed{8}7\boxed{4}7 \end{array}$$

Add

$$\begin{array}{r} \boxed{4}628 \\ +\ \boxed{7}7\boxed{6} \\ \hline 54\boxed{0}4 \end{array}$$

Add

$$\begin{array}{r} \boxed{2}369 \\ 39 \\ +\ \boxed{5}6\boxed{6} \\ \hline 29\boxed{7}4 \end{array}$$

Subtract

$$\begin{array}{r} 7\boxed{8}9 \\ -\boxed{3}63 \\ \hline 42\boxed{6} \end{array}$$

Subtract

$$\begin{array}{r} \boxed{3}8\boxed{2}66 \\ -\ 8\boxed{3}9 \\ \hline 3\boxed{7}41\boxed{7} \end{array}$$

Subtract

$$\begin{array}{r} \boxed{8}2\boxed{1} \\ -295 \\ \hline 5\boxed{2}6 \end{array}$$

5

Put each number on the display of a hand-calculator using just these keys:

$\boxed{5}, \boxed{6}, \boxed{8}, \boxed{9}, \boxed{+}, \boxed{-}, \boxed{\times}, \boxed{\div}, \boxed{=}$

Write the keys in the order you use them. You may use a key more than once.

It costs 1¢ each time you press a key. Try to spend less than the amount shown for each number.

$\boxed{8}\boxed{-}\boxed{6}\boxed{+}\boxed{5}\boxed{=}$ ________ 7 [9¢]

$\boxed{9}\boxed{9}\boxed{-}\boxed{5}\boxed{5}\boxed{=}$ ________ 44 [10¢]

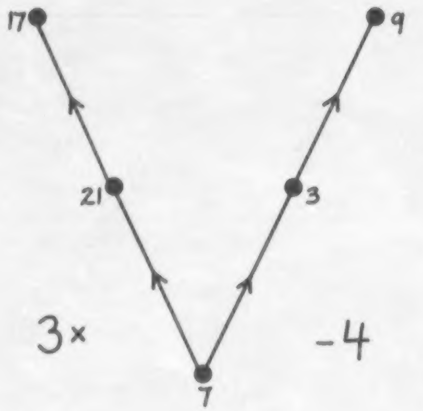
$\boxed{5}\boxed{\times}\boxed{8}\boxed{\times}\boxed{5}\boxed{=}$ ________ 200 [12¢]

$\boxed{5}\boxed{-}\boxed{5}\boxed{=}\boxed{=}\boxed{=}$ ________ ⁻10 [10¢]

(Many solutions are possible. Refer to Section 2(f) of the General Introduction for a discussion of hand-calculator characteristics which could effect the solutions.)

Label each dot with one of these numbers.

3 7 9 17 21



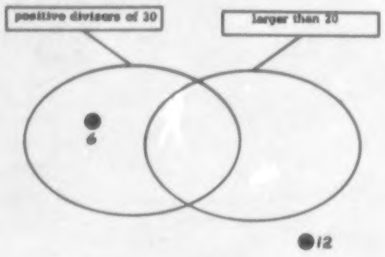
The red tag is one of these:

- ~~[illegible]~~
- ~~[illegible]~~
- ~~[illegible]~~
- ~~[illegible]~~
- ~~[illegible]~~
- ~~[illegible]~~
- positive divisors of 30

The blue tag is one of these:

- ~~[illegible]~~
- ~~[illegible]~~
- ~~[illegible]~~
- ~~[illegible]~~
- larger than 20
- ~~[illegible]~~
- ~~[illegible]~~

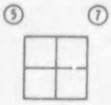
Label the strings.



Mup is a secret number.

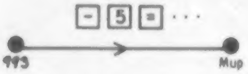
First clue

Mup can be put on the ones' board of the Minicomputer with exactly one of these checkers.



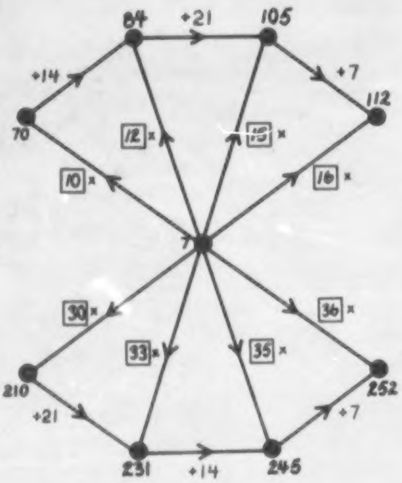
Mup could be 5, 10, 20, 40, 7, 14, 28, or 56.

Second clue

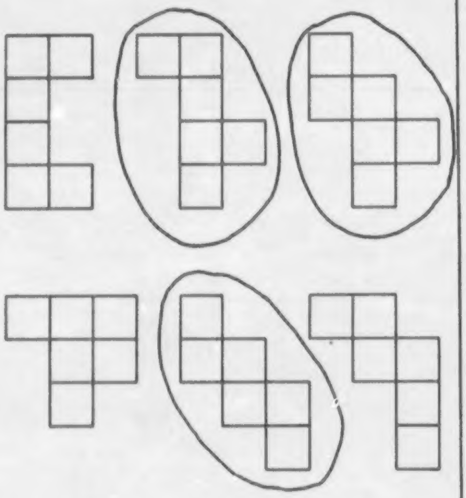


Mup is 28.

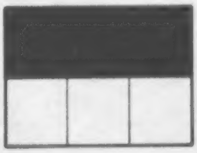
Label the dots and fill in the boxes for the red arrows.


84
+21
105
+14
+7
112
70
12 x
15 x
10 x
16 x
7
30 x
36 x
33 x
35 x
210
252
+21
+7
231
+14
245

Circle each of the shapes which can be folded into a cube.

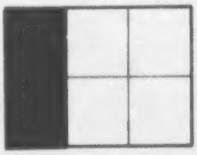


Color one-half of this region
red and fill in the box.



$\frac{1}{2} = \frac{\boxed{3}}{6}$

Color one-third of this region
red and fill in the box.



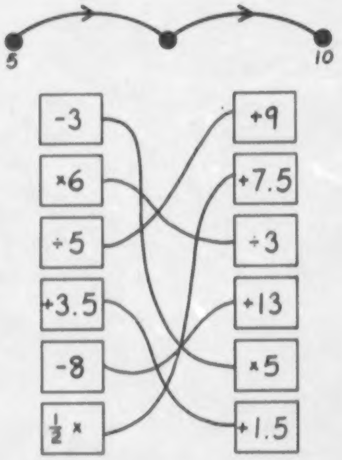
$\frac{1}{3} = \frac{\boxed{2}}{6}$

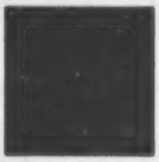
Complete:

$\frac{1}{2} + \frac{1}{3} = \frac{\boxed{3}}{6} + \frac{\boxed{2}}{6} = \underline{\frac{5}{6}}$

(Other colorings are possible.)

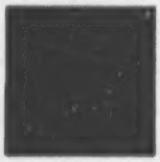
Pair the tags.

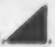

5
10
-3
×6
÷5
+3.5
-8
½ ×
+9
+7.5
÷3
+13
×5
+1.5

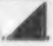


How many triangles of this size, , could fit into the red region? 8

How many triangles of this size, , could fit into the blue region? 8



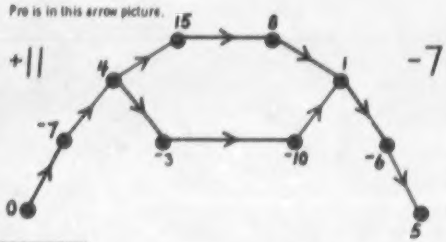
How many triangles of this size,  , could fit into the red region? 19

How many triangles of this size,  , could fit into the blue region? 13

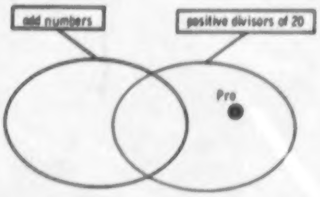
Pro is a secret number.

First clue

Pro is in this arrow picture.

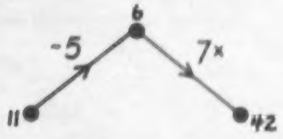


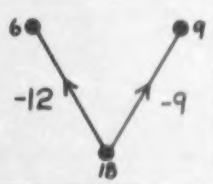
Second clue

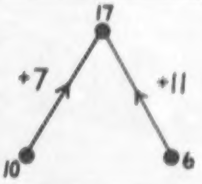


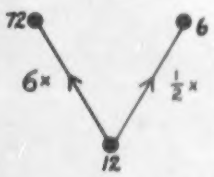
Pro is 4.

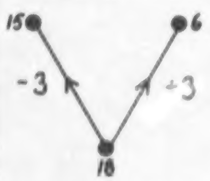
6 is the smallest number in each arrow picture. Find 6 in each picture. Label all the dots.











Put one of the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in each box to make the calculations correct. You may use a digit more than once.

Subtract

$$\begin{array}{r} \boxed{3}\,8\,\boxed{8}\,4\,\boxed{5} \\ -\quad 2\,\boxed{7}\,6 \\ \hline 3\,\boxed{8}\,5\,6\,9 \end{array}$$

Subtract

$$\begin{array}{r} 6\,0\,4 \\ -\boxed{1}\,5\,\boxed{8} \\ \hline 4\,\boxed{4}\,6 \end{array}$$

Multiply

$$\begin{array}{r} \boxed{2}\,7\,\boxed{6} \\ \times\quad 9 \\ \hline 2\,4\,\boxed{8}\,4 \end{array}$$

Multiply

$$\begin{array}{r} \boxed{7}\,5\,7 \\ \times\quad \boxed{6} \\ \hline 4\,\boxed{5}\,\boxed{4}\,2 \end{array}$$

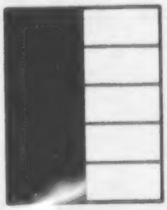
Divide

$$\begin{array}{rr|r} & 3\,5\,8 & R=2 \\ 8\,) & 2\,8\,6\,6 & \\ & -2\,4\,0\,0 & 300 \\ \hline & 4\,6\,6 & \\ & -4\,0\,0 & 50 \\ \hline & 6\,6 & \\ & -6\,4 & 8 \\ \hline & 2 & \end{array}$$

Divide

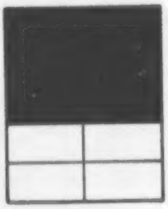
$$\begin{array}{rr|r} & \boxed{2}\,\boxed{3}\,\boxed{9} & R=\boxed{1} \\ 4\,) & 9\,5\,\boxed{7} & \\ & -\boxed{8}\,\boxed{0}\,\boxed{0} & \boxed{2}\,0\,0 \\ & \boxed{1}\,\boxed{5}\,7 & \\ & -\boxed{1}\,\boxed{2}\,\boxed{0} & \boxed{3}\,0 \\ \hline & \boxed{3}\,\boxed{7} & \\ & -\boxed{3}\,\boxed{6} & \boxed{9} \\ \hline & \boxed{1} & \end{array}$$

Color one-half of this region red and fill in the box.



$\frac{1}{2} = \frac{\boxed{5}}{10}$

Color three-fifths of this region red and fill in the box.



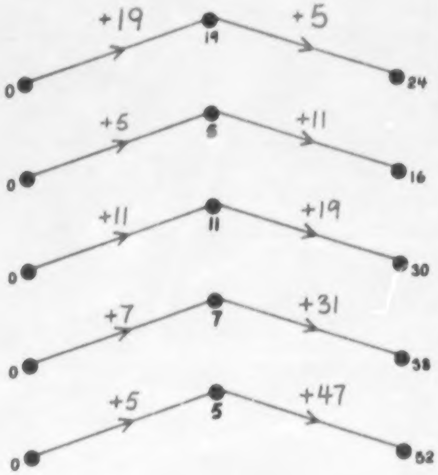
$\frac{3}{5} = \frac{\boxed{6}}{10}$

Complete:

$\frac{1}{2} + \frac{3}{5} = \underline{\frac{11}{10} = 1\frac{1}{10}}$

(Other colorings are possible. Different names for $\frac{11}{10}$ could be given.)

In these six arrow roads, label each blue arrow "+☐" with a PRIME number in the box. The first road is done for you. Many solutions are possible.


+19
+5
0
19
24
+5
+11
0
5
16
+11
+19
0
11
30
+7
+31
0
7
38
+5
+47
0
5
52

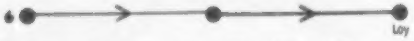
(Other solutions are possible.)

Lay is a secret number.

First clue

The BLUE arrow could be for:
5×
÷2
+7

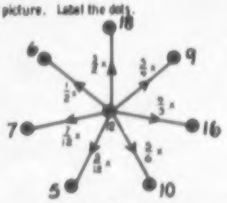
The RED arrow could be for:
5×
÷2
+7



Lay could be 1.5, 6.5, 10, 15, 20, 37, 65, or 150.

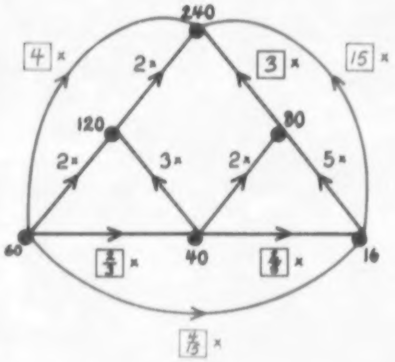
Second clue

Lay is in this arrow picture. Label the dots.


18
9
16
10
5
7
6
12
$\frac{3}{2}$×
$\frac{3}{4}$×
$\frac{4}{3}$×
$\frac{5}{6}$×
$\frac{5}{12}$×
$\frac{7}{12}$×
$\frac{1}{2}$×

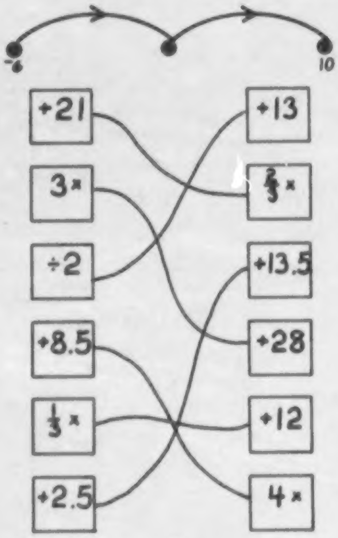
Lay is 10.

Label the dots and fill in the boxes for the arrows.


240
120
80
60
40
16
4 ×
2×
3 ×
15 ×
2×
3×
2×
5×
$\frac{2}{3}$ ×
$\frac{2}{5}$ ×
$\frac{4}{15}$ ×

(Other labels for the arrows could be given; e.g., $\frac{2}{3}$x = $\frac{4}{6}$x = $\frac{8}{12}$x = ...)

Pair the tags.

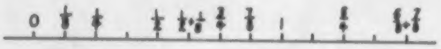


⁻6
10
+21
3×
÷2
+8.5
1/3 ×
+2.5
+13
2/3 ×
+13.5
+28
+12
4×

22

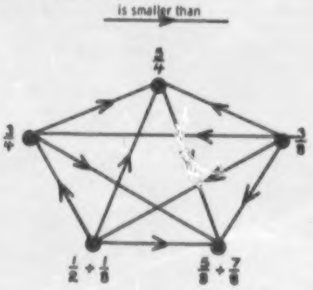
On the number line, locate these numbers:

$\frac{1}{2}$; $\frac{1}{8}$; $\frac{7}{8}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{5}{4}$; $\frac{5}{8}+\frac{7}{8}$; $\frac{1}{2}+\frac{1}{8}$.



0
1/8
1/4
1/2
1/2 + 1/8
3/4
7/8
1
5/4
5/8 + 7/8

Draw all the missing red arrows.



is smaller than
5/4
3/4
3/8
1/2 + 1/8
5/8 + 7/8

23

Sid is a secret number.

First clue

Sid can be written by using each of these symbols exactly once.

0.3 +

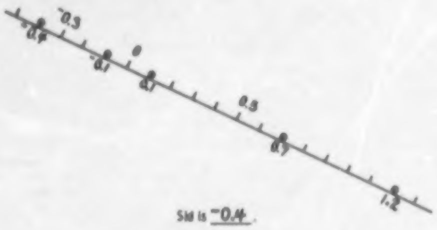
(1.4 −

0.7)

Sid could be ⁻1.8, ⁻1.0, ⁻0.4, 0.4, 1.0, or 1.8.

Second clue

Sid is one of these red dots. Label the dots.



⁻0.3
0
⁻0.1
0.1
0.5
0.7
1.2

Sid is ⁻0.4.

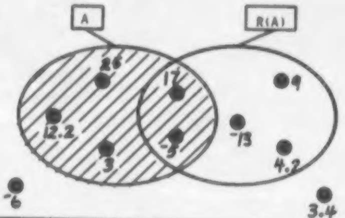
24

Put each of these numbers in the string picture.

⁻13 ⁻5 3.4 9 17

⁻6 3 4.2 12.2 25

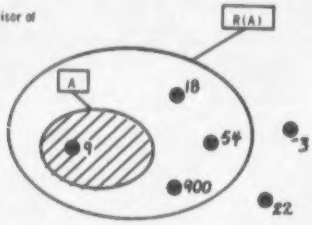
R = −8



A
R(A)
25
17
12.2
3
⁻5
9
⁻13
4.2
⁻6
3.4

Label the dots. Many answers are possible.

R= is a positive divisor of



R(A)
A
18
9
54
900
⁻3
22

25

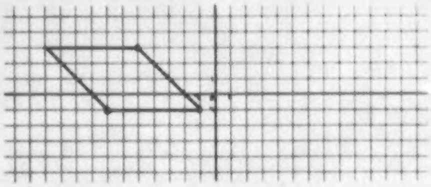
(Many solutions are possible.)

Three corners of a parallelogram are at (¯7, ¯1), (¯5, 2), and (¯1, ¯1).

Draw dots at these three corners. (¯11, 3) or

Where could the fourth corner be? (1, 3) or (¯3, ¯5) Several answers are possible.

Then draw one of these parallelograms.



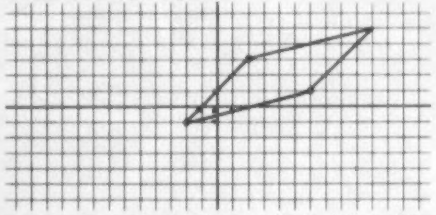
(The parallelogram shown has the fourth corner at (¯11, 3). Another parallelogram could be drawn with the fourth corner at (1, 3) or at (¯3, ¯5).)

Three corners of a parallelogram are at (¯2, ¯1), (2, 3), and (6, 1).

Draw dots at these three corners. (10, 5) or (2, ¯3) or

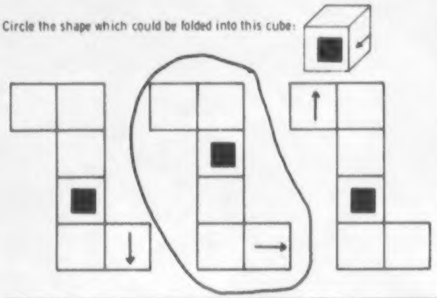
Where could the fourth corner be? (¯6, 1) Several answers are possible.

Then draw one of these parallelograms.

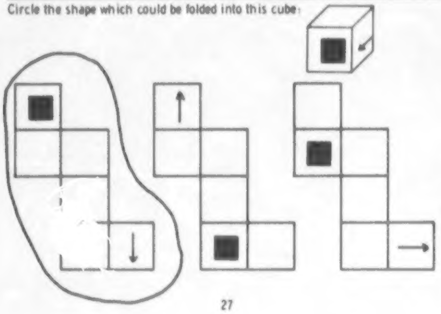


(The parallelogram shown has the fourth corner at (10, 5). Another parallelogram could be drawn with the fourth corner at (2, ¯3) or at (¯6, 1).)

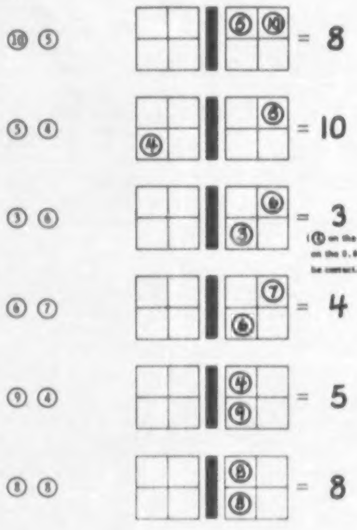
Circle the shape which could be folded into this cube:



Circle the shape which could be folded into this cube:



Put each number on the Minicomputer using exactly the checkers to the left of the boards.



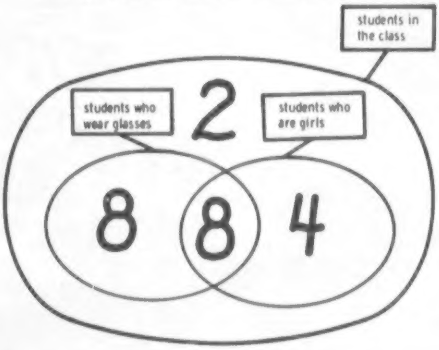
(⑥ on the 0.1-square and ③ on the 0.8-square would also be correct.)

There are 22 students in this class.

12 of the students in the class are girls.

One-third of the girls in the class do not wear glasses.

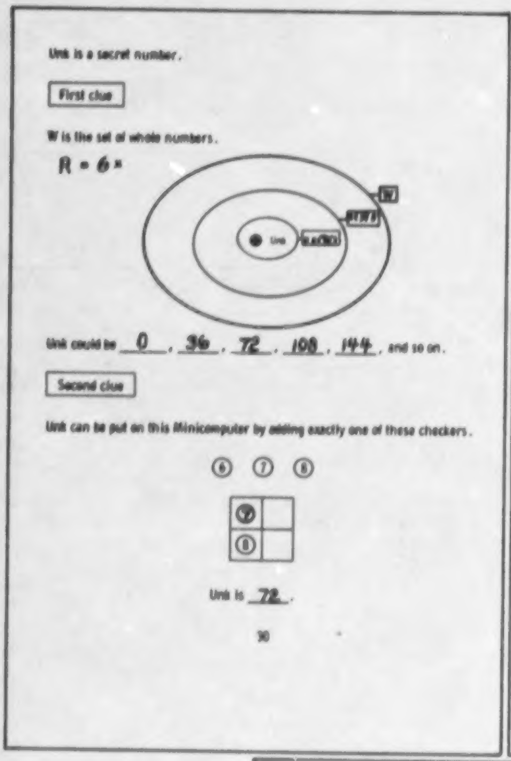
One-half of the students in the class who wear glasses are boys.



How many of the girls in the class wear glasses? 8

How many of the boys in the class do not wear glasses? 2

How many of the students in the class wear glasses? 16



Unk is a secret number.

First clue

W is the set of whole numbers.

R = 6×

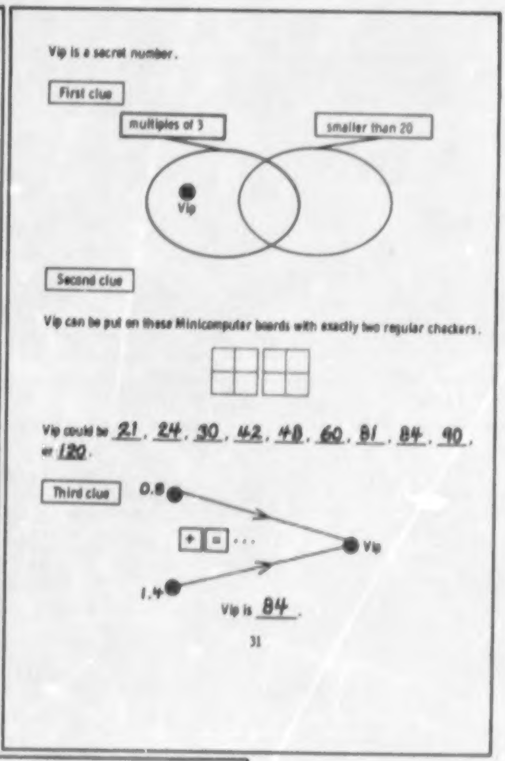
Unk could be 0, 36, 72, 108, 144, and so on.

Second clue

Unk can be put on this Minicomputer by adding exactly one of these checkers.

Unk is 72.

30



Vip is a secret number.

First clue

multiples of 3
smaller than 20
Vip

Second clue

Vip can be put on these Minicomputer boards with exactly two regular checkers.

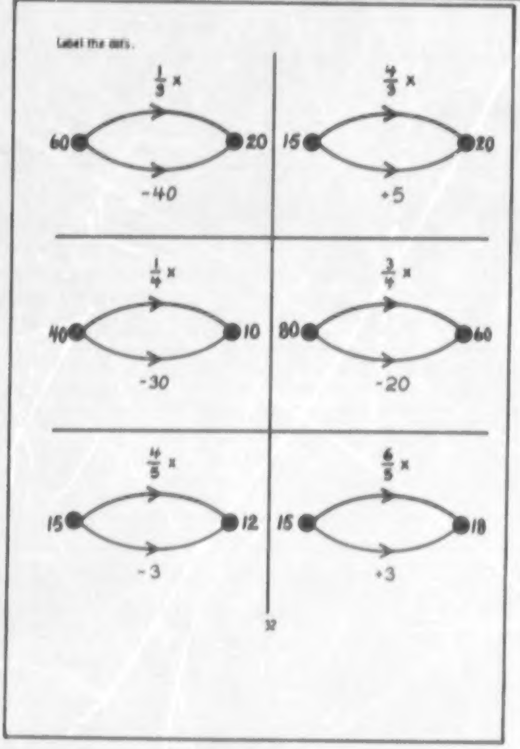
Vip could be 21, 24, 30, 42, 48, 60, 81, 84, 90, or 120.

Third clue

0.8
+
=
...
Vip
1.4

Vip is 84.

31



Label the dots.

$\frac{1}{3}\times$
60
20
−40
$\frac{4}{3}\times$
15
20
+5
$\frac{1}{4}\times$
40
10
−30
$\frac{3}{4}\times$
80
60
−20
$\frac{4}{5}\times$
15
12
−3
$\frac{6}{5}\times$
15
18
+3

32

A
SELECTION
OF
PROBLEMS
#6

A SELECTION OF PROBLEMS #6

The problems in this workbook are in order of increasing difficulty. Approximately the first ten pages of the workbook have easy problems, the next ten to twelve pages have average problems, and the last ten pages have more challenging problems. It is suggested that all students work through as many pages as they can handle during the two lessons scheduled for the workbook. Not all students are expected to complete the entire workbook; some will succeed to do only the easy problems, many should finish about twenty pages, and only the faster students will reach the most challenging problems at the end of the booklet.

Many problems in the workbook were designed with the availability of hand-calculators in mind. If possible, provide all the students with hand-calculators or at least give students access to hand-calculators.[#] You will find that many students do not need or want to use calculators; however, some students still need the support of the calculator in thinking about these problems.

A brief collective lesson is suggested for the first workbook period. This should be a short warm-up activity serving to review a situation similar to one in the workbook that might cause difficulty.

[#] See Section 2(f) of the General Introduction for additional comments on the use of hand-calculators and on desirable characteristics of hand-calculators.

W 15

Lesson 1

Provide each student with a hand-calculator.

Write this information on the board.

$\boxed{3}\quad\boxed{7}\quad\boxed{+}\quad\boxed{-}\quad\boxed{\times}\quad\boxed{\div}\quad\boxed{=}$

T: Using only these keys, put 84 on the display of your hand-calculator. On a piece of paper, write the keys in the order you use them. Each time you press a key, it costs one cent. Try to spend less than fifteen cents.

Let the students work independently for several minutes as you observe their work. Encourage students to find several solutions.

Invite several students to announce their solutions to the class and write each solution on the board as it is presented. A few of the many possible solutions and their prices are presented below.

$\boxed{7}\ \boxed{7}\ \boxed{+}\ \boxed{7}\ \boxed{=}$ 84 [5¢]

$\boxed{3}\ \boxed{+}\ \boxed{7}\ \boxed{\times}\ \boxed{7}\ \boxed{+}\ \boxed{7}\ \boxed{=}\ \boxed{=}$ 84 [9¢]

$\boxed{3}\ \boxed{\times}\ \boxed{3}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{+}\ \boxed{3}\ \boxed{=}$ 84 [9¢]

$\boxed{3}\ \boxed{+}\ \boxed{3}\ \boxed{+}\ \boxed{7}\ \boxed{\times}\ \boxed{7}\ \boxed{-}\ \boxed{7}\ \boxed{=}$ 84 [10¢]

Repeat this activity asking the students to display $^{-}1$. Several solutions for $^{-}1$ are presented below.

$\boxed{3}\ \boxed{-}\ \boxed{7}\ \boxed{+}\ \boxed{3}\ \boxed{=}$ $^{-}1$ [6¢]

$\boxed{3}\ \boxed{3}\ \boxed{-}\ \boxed{3}\ \boxed{7}\ \boxed{+}\ \boxed{3}\ \boxed{=}$ $^{-}1$ [8¢]

$\boxed{3}\ \boxed{-}\ \boxed{3}\ \boxed{=}\ \boxed{=}\ \boxed{\div}\ \boxed{3}\ \boxed{=}$ $^{-}1$ [8¢]

$\boxed{3}\ \boxed{\times}\ \boxed{3}\ \boxed{-}\ \boxed{7}\ \boxed{-}\ \boxed{3}\ \boxed{=}$ $^{-}1$ [8¢]

Give each student a copy of the workbook "A Selection of Problems #6" and let them work on their own.

If you observe many students struggling with the same problem (or part of a problem), you could have a short collective discussion about that problem.

At the end of this lesson, collect the workbooks and have them ready to return for Lesson 2.

W 16

Lesson 2

Return the students' copies of "A Selection of Problems #6". You may wish to discuss any page that was difficult for most of the students. Then ask the students to correct their mistakes, if any, and to continue working in their workbooks.

If you observe many students struggling with the same problem (or part of a problem), you could have a short discussion about that problem.

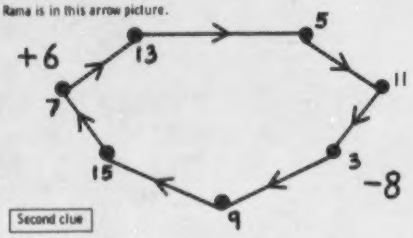
At the end of this lesson, collect the workbooks for your progress check. You may wish to let some students work further in their workbooks later during any free study time.

A SELECTION OF PROBLEMS #6

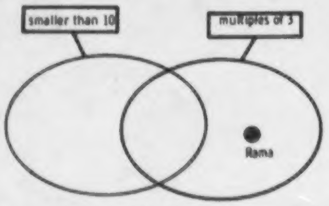
Rama is a secret number.

First clue

Rama is in this arrow picture.

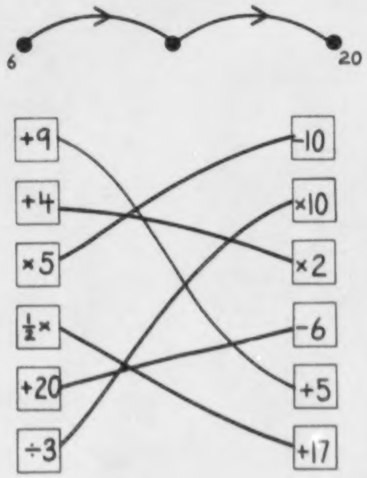

+6
13
5
11
3
-8
9
15
7

Second clue

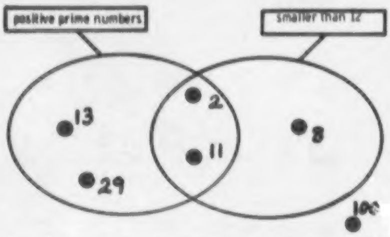

smaller than 10
multiples of 3
Rama

Rama is 15.

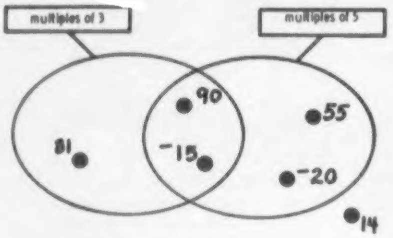
Pair the tags. One is done for you.


6
20
+9
+4
×5
½×
+20
÷3
-10
×10
×2
-6
+5
+17

Label the dots. Many answers are possible.

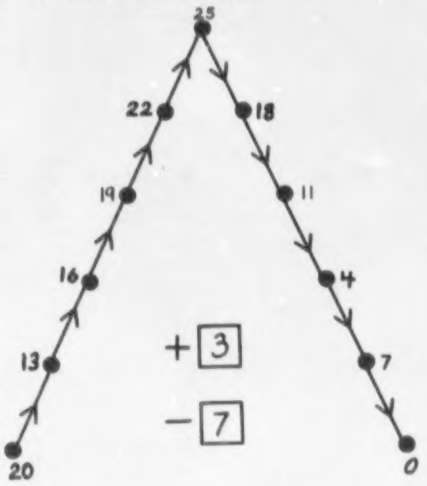

positive prime numbers
smaller than 12
13
29
2
11
8
100

Label the dots. Many answers are possible.


multiples of 3
multiples of 5
81
90
-15
55
-20
14

(Many solutions are possible.)

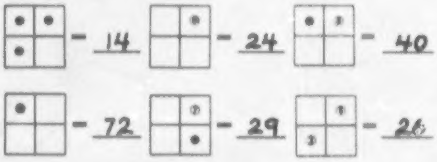
Fill in the boxes for the arrows and label the dots.


25
22
19
16
13
20
18
11
4
7
0
+ 3
− 7

Bof is a secret number.

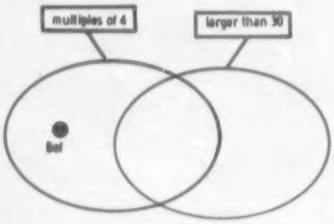
First clue

Bof is one of these numbers.


– 14   – 24   – 40

– 72   – 29   – 26

Second clue



Bof is 24.

Put one of the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in each box to make the calculations correct. You may use a digit more than once.

Add

$$\begin{array}{r} \boxed{6}\,6\,\boxed{4}\,8\,\boxed{0} \\ +\ \boxed{4}\,5\,9\,2 \\ \hline 7\,1\,0\,\boxed{7}\,2 \end{array}$$

Add

$$\begin{array}{r} 4\,6\,\boxed{6} \\ 5\,\boxed{4}\,3\,8 \\ +\ 6\,7 \\ \hline \boxed{5}\,9\,\boxed{7}\,1 \end{array}$$

Add

$$\begin{array}{r} 6{,}0\,8\,8 \\ 5\,1\,8 \\ +\ 7{,}1\,9\,8 \\ \hline 13{,}804 \end{array}$$

Subtract

$$\begin{array}{r} \boxed{6}\,9\,8 \\ -\ 2\,\boxed{9}\,\boxed{3} \\ \hline 4\,0\,5 \end{array}$$

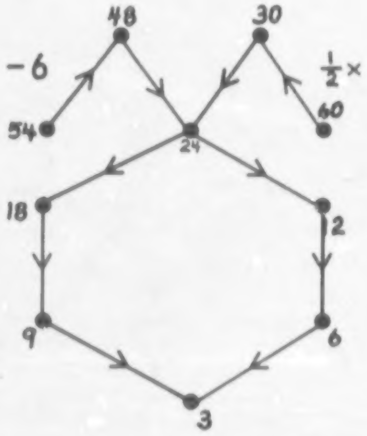
Subtract

$$\begin{array}{r} 4\,\boxed{5}\,8\,2 \\ -\ 7\,\boxed{4}\,5 \\ \hline \boxed{3}\,8\,3\,\boxed{7} \end{array}$$

Subtract

$$\begin{array}{r} 16{,}2\,4\,0 \\ -\ 9\,3\,7 \\ \hline 15{,}303 \end{array}$$

Label the dots.



In this picture, exactly three numbers are multiples of 9.

They are 9, 18, and 54.

Put each number on the display of a hand-calculator using just these keys:

$\boxed{4}$, $\boxed{5}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, $\boxed{=}$.

Write the keys in the order you use them. You may use a key more than once.

$\boxed{5}\boxed{\times}\boxed{4}\boxed{+}\boxed{4}\boxed{-}\boxed{5}\boxed{=}$ 21

$\boxed{5}\boxed{5}\boxed{+}\boxed{5}\boxed{=}$ 60

$\boxed{4}\boxed{-}\boxed{5}\boxed{=}\boxed{=}$ ⁻6

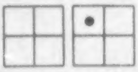
$\boxed{5}\boxed{-}\boxed{=}\boxed{=}\boxed{\times}\boxed{4}\boxed{=}$ ⁻20

(Many solutions are possible. Refer to Section 2(f) of the General Introduction for a discussion of hand-calculator characteristics which could effect the solutions.)

Sle is a secret number.

First clue

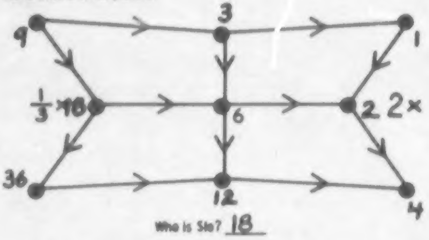
Sle can be put on this Minicomputer by adding exactly one ⑤-checker.



Sle could be 13, 18, 28, 48, 58, 108, 208 or 408.

Second clue

Sle is in this arrow picture.


3
9
1
$\frac{1}{3}$ × 18
6
2
2×
36
12
4

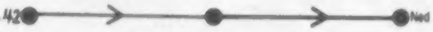
Who is Sle? 18

Ned is a secret number.

First clue

The BLUE arrow could be for:
+6
−6
÷6

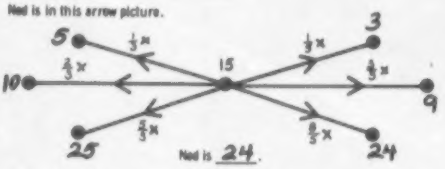
The RED arrow could be for:
−20
÷2



Ned could be ⁻13, 3.5, 16, 18, 24, or 28.

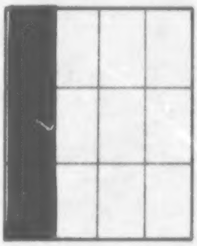
Second clue

Ned is in this arrow picture.


5
$\frac{1}{3}$×
$\frac{1}{5}$×
3
15
10
$\frac{2}{3}$×
$\frac{3}{5}$×
9
$\frac{5}{3}$×
$\frac{8}{5}$×
25
24

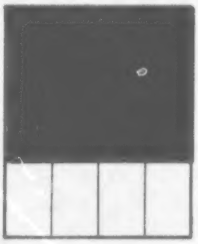
Ned is 24.

Color one-fourth of this region blue and fill in the box.



$\frac{1}{4} = \frac{\boxed{3}}{12}$

Color two-thirds of this region blue and fill in the box.



$\frac{2}{3} = \frac{\boxed{8}}{12}$

Complete.

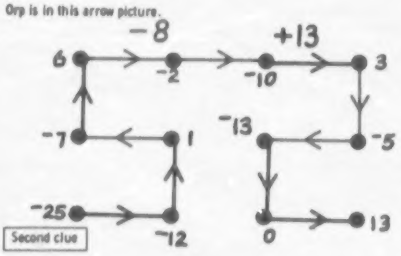
$\frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$

(Other colorings are possible.)

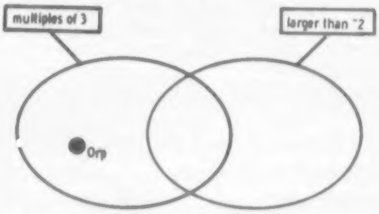
Orp is a secret number.

First clue

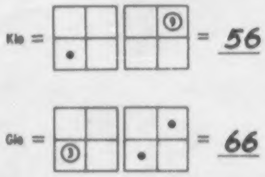
Orp is in this arrow picture.


−8
+13
6
⁻2
⁻10
3
⁻7
1
⁻13
⁻5
⁻25
⁻12
0
13

Second clue

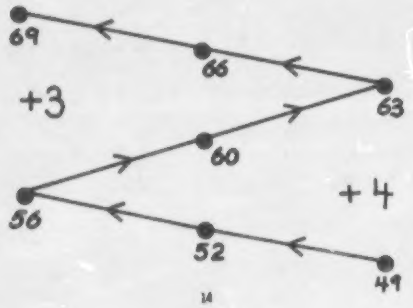

multiples of 3
larger than ⁻2
Orp

Who is Orp? ⁻12



Kio =
= 56
Gio =
= 66

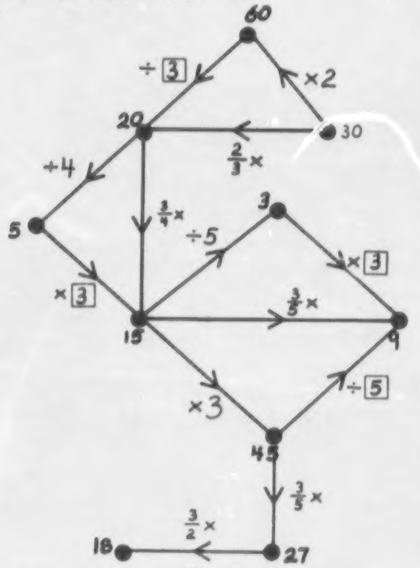
Kio and Gio are in this arrow picture.
Find the dots for Kio and Gio. Then label all the dots.



69
66
+3
63
60
+4
56
52
49

14

Label the dots and fill in the boxes for the arrows.



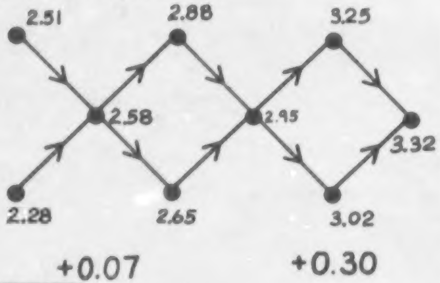
60
÷ 3
×2
20
30
2/3 ×
+4
3/4 ×
3
÷5
5
× 3
× 3
3/5 ×
15
9
÷ 5
×3
45
3/5 ×
3/2 ×
18
27

15

Simo is a secret number.

First clue

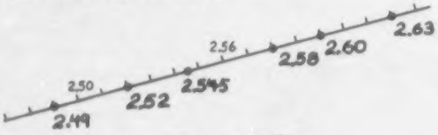
Simo is in this arrow picture.



2.51
2.88
3.25
2.58
2.95
3.32
2.28
2.65
3.02
+0.07
+0.30

Second clue

Simo is one of these red dots. Label the dots.



2.56
2.58
2.60
2.63
2.50
2.545
2.52
2.49

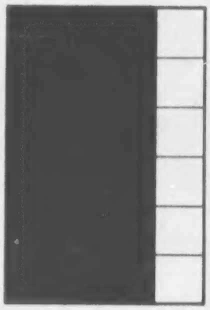
Who is Simo? 2.58

16

Color five-sixths of this region red and fill in the box.

$\frac{5}{6} = \frac{\boxed{20}}{24}$

Color three-fourths of this region red and fill in the box.



$\frac{3}{4} = \frac{\boxed{18}}{24}$

Complete.

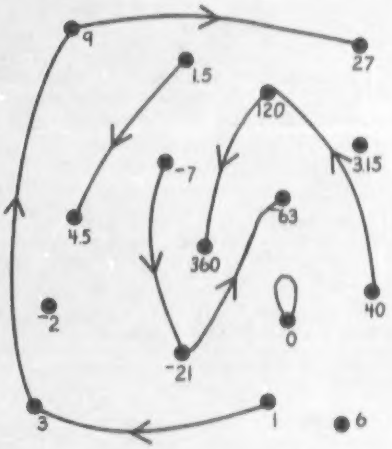
$\frac{5}{6} + \frac{3}{4} = \frac{38}{24}$

17

(Other colorings are possible. Different names for $\frac{38}{24}$ could be given; e.g., $\frac{38}{24} = 1\,\frac{14}{24} = \frac{19}{12} = 1\,\frac{7}{12}$.)

Draw as many red arrows as you can in this picture.

3×



Put one of the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in each box to mak the calculations correct. You may use a digit more than once.

Subtract

$$\begin{array}{r} 3\,0\,7\,\boxed{6}\,\boxed{0} \\ -\,6\,\boxed{8}\,2\,6 \\ \hline \boxed{2}\,\boxed{3}\,9\,3\,4 \end{array}$$

Subtract

$$\begin{array}{r} \boxed{8}\,5\,0\,5 \\ -\,3\,\boxed{0}\,\boxed{7}\,8 \\ \hline 5\,4\,2\,\boxed{7} \end{array}$$

Multiply

$$\begin{array}{r} \boxed{8}\,7\,6 \\ \times\quad \boxed{5} \\ \hline 4\,3\,\boxed{8}\,0 \end{array}$$

Multiply

$$\begin{array}{r} \boxed{6}\,7 \\ \times\,\boxed{8}\,6 \\ \hline 4\,0\,\boxed{2} \\ \boxed{5}\,\boxed{3}\,6\,0 \\ \hline \boxed{5}\,\boxed{7}\,\boxed{6}\,\boxed{2} \end{array}$$

Divide

$$\begin{array}{rr|r} & \boxed{2}\,\boxed{1}\,\boxed{6} \;\; R=\boxed{2} & \\ 12\,) & 2\,5\,9\,\boxed{4} & \\ & -\boxed{2}\,\boxed{4}\,\boxed{0}\,\boxed{0} & \boxed{2}\,0\,0 \\ \hline & \boxed{1}\,\boxed{9}\,\boxed{4} & \\ & -\,\boxed{1}\,\boxed{2}\,\boxed{0} & \boxed{1}\,0 \\ \hline & \boxed{7}\,4 & \\ & -\,\boxed{7}\,\boxed{2} & \boxed{6} \\ \hline & \boxed{2} & \end{array}$$

Divide

$$\begin{array}{rr|r} & 3\,0\,7 \;\; R=0 & \\ 15\,) & 4{,}6\,0\,5 & \\ & -4\,5\,0\,0 & 300 \\ \hline & 1\,0\,5 & \\ & -1\,0\,5 & 7 \\ \hline & 0 & \end{array}$$

Put each number on the hand-calculator using just these keys:

$\boxed{4}$, $\boxed{5}$, $\boxed{7}$, $\boxed{8}$, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, $\boxed{=}$.

Write the keys in the order you use them. You may use a key more than once.

It costs 1¢ each time you press a key. Try to spend less than the amount shown for each number.

$\boxed{4}\boxed{5}\boxed{-}\boxed{5}\boxed{5}\boxed{=}$ ⁻10 [9¢]

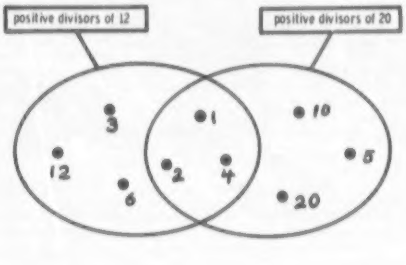
$\boxed{8}\boxed{-}\boxed{4}\boxed{5}\boxed{+}\boxed{5}\boxed{=}$ ⁻32 [10¢]

$\boxed{4}\boxed{\div}\boxed{8}\boxed{+}\boxed{=}\boxed{=}$ 1.5 [11¢]

$\boxed{4}\boxed{\div}\boxed{4}\boxed{\div}\boxed{5}\boxed{+}\boxed{8}\boxed{-}\boxed{5}\boxed{=}$ 3.2 [13¢]

(Many solutions are possible. Refer to Section 2(f) of the General Introduction for a discussion of hand-calculator characteristics which could effect the solutions.)

Put all the positive divisors of 12 and of 20 in this string picture.



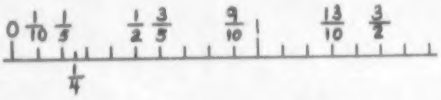
Complete.

$\sqcap$: Greatest Common Divisor

12 ⊓ 20 = 4
12 ⊓ 30 = 6
26 ⊓ 20 = 2

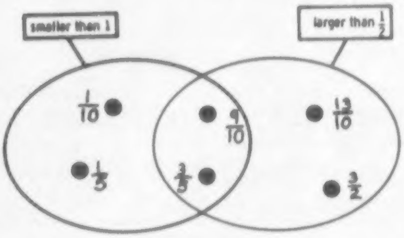
Locate these numbers on the number line.

$\frac{1}{10}$ $\frac{9}{10}$ $\frac{13}{10}$ $\frac{1}{5}$ $\frac{3}{5}$ $\frac{1}{2}$ $\frac{3}{2}$ $\frac{1}{4}$


0
1/10
1/5
1/2
3/5
9/10
1
13/10
3/2
1/4

Put each of these numbers in the string picture.

$\frac{1}{10}$ $\frac{9}{10}$ $\frac{13}{10}$ $\frac{1}{5}$ $\frac{3}{5}$ $\frac{3}{2}$


smaller than 1
larger than 1/2
1/10
1/5
9/10
3/5
13/10
3/2

22

Lada is a secret number.

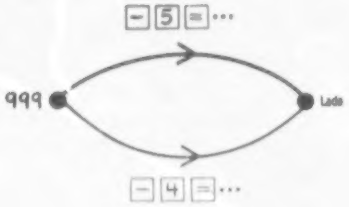
First clue

Lada can be written by adding parentheses to this expression.

$$3 \times 5 - 2 \times 8$$

Lada could be $^{-}33$, $^{-}1$, 72, or 104.

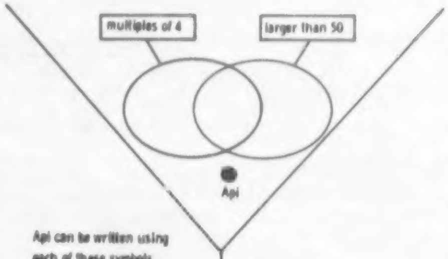
Second clue



Lada is $^{-}1$.

23

Api is a secret number.


multiples of 4
larger than 50
Api

Api can be written using
each of these symbols
exactly once.

× 6
(+ 7
2)

Api $\equiv_3$ 8

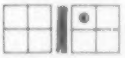
Api is 26.

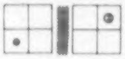
24

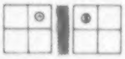
Han is a secret number.

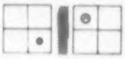
First clue

Han is one of these numbers.


= 4.8

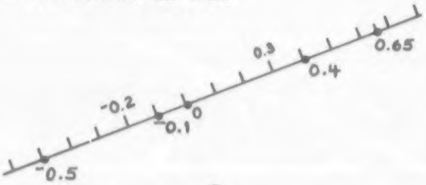

= 3.2


= 0


= 0.2

Second clue

Han is one of these red dots. Label the dots.


-0.5
-0.2
-0.1
0
0.3
0.4
0.65

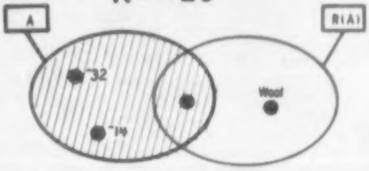
Han is 0.

25

Woof is a secret number.

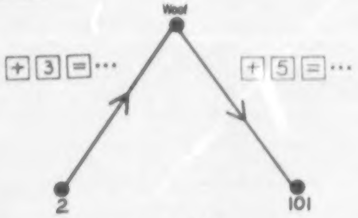
First clue

R = +20


A
R(A)
$^{-}32$
$^{-}14$
Woof

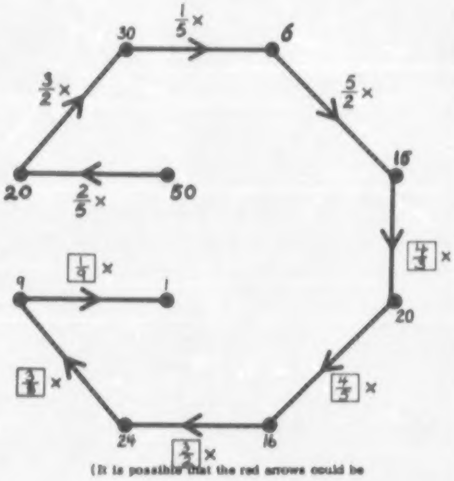
Woof could be $^{-}12$, 6, 8, or 26.

Second clue


Woof
+ 3 = ···
+ 5 = ···
2
101

Woof is 26.

Label the dots and fill in the boxes for the arrows.


30
$\frac{1}{5}$ ×
6
$\frac{3}{2}$ ×
$\frac{5}{2}$ ×
15
20
$\frac{2}{5}$ ×
50
$\frac{4}{3}$ ×
$\frac{1}{9}$ ×
9
1
20
$\frac{3}{8}$ ×
$\frac{4}{5}$ ×
24
16
$\frac{3}{2}$ ×

(It is possible that the red arrows could be labeled with other names for the numbers; e.g., $\frac{4}{3} = \frac{8}{6} = \frac{12}{9} = \frac{16}{12} = \frac{20}{15} = \ldots$.)

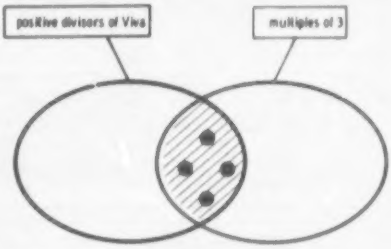
Viva is a secret number.

First clue

60 is the least common multiple of 12 and Viva.

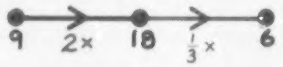
Viva could be 5, 10, 15, 20, 30, or 60.

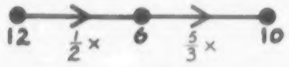
Second clue

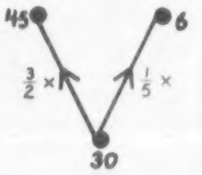

positive divisors of Viva
multiples of 3

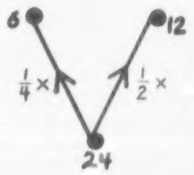
Viva is 30.

6 is the smallest number in each arrow picture. Find 6 in each picture. Label all the dots.


9
2×
18
$\frac{1}{3}$ ×
6


12
$\frac{1}{2}$ ×
6
$\frac{5}{3}$ ×
10


45
6
$\frac{3}{2}$ ×
$\frac{1}{5}$ ×
30


6
12
$\frac{1}{4}$ ×
$\frac{1}{2}$ ×
24

In each table, write a "T" for each true number story. For a false number story, leave the space in the table blank. One true number story in each table is marked for you.

$<$	1	$\frac{1}{3}$	0.75	$\frac{3}{2}$
$\frac{3}{4}$	T			T
0.4	T		T	T
1.2				T
$\frac{5}{4}$				T

$=_3$	$^-36$	5	29	100
$^-5$				T
0	T			
26		T	T	

Put one of the digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 in each box to make the calculations correct. You may use a digit more than once.

Subtract

$$\begin{array}{r} 5\boxed{0}\boxed{2}6 \\ -\ 49\boxed{8} \\ \hline \boxed{4}528 \end{array}$$

Subtract

$$\begin{array}{r} \boxed{6}4\boxed{0} \\ -\ 241 \\ \hline 3\boxed{9}9 \end{array}$$

Multiply

$$\begin{array}{r} \boxed{6}\boxed{7} \\ \times\ \boxed{5}2 \\ \hline 1\boxed{3}4 \\ \boxed{3}\boxed{3}50 \\ \hline \boxed{3}\boxed{4}8\boxed{4} \end{array}$$

Multiply

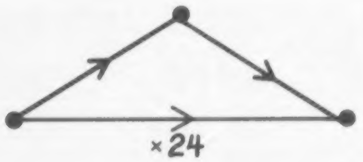
$$\begin{array}{r} \boxed{6}\boxed{7}\boxed{9} \\ \times\ \boxed{5}\boxed{3} \\ \hline \boxed{2}\boxed{0}3\boxed{7} \\ \boxed{3}39\boxed{5}0 \\ \hline 3\boxed{5}\boxed{9}87 \end{array}$$

Multiply

$$\begin{array}{r} \boxed{4}\boxed{8} \\ \times\ \boxed{8}9 \\ \hline \boxed{4}3\boxed{2} \\ \boxed{3}8\boxed{4}0 \\ \hline \boxed{4}\boxed{2}72 \end{array}$$

Divide

$$\begin{array}{rr|r} & \boxed{1}3\boxed{3} & n=\boxed{8} \\ 2\boxed{7}\,) & 3\boxed{5}\boxed{9}9 & \\ & -\ \boxed{2}7\boxed{0}\boxed{0} & \boxed{1}00 \\ \hline & 8\boxed{9}\boxed{9} & \\ & -\ \boxed{8}\boxed{1}\boxed{0} & \boxed{3}0 \\ \hline & 8\boxed{9} & \\ & -\ \boxed{8}\boxed{1} & \boxed{3} \\ \hline & \boxed{8} & \end{array}$$



Complete the charts.

→	→
×4	×6
×2	×12
×3	× 8
÷2	×48
×72	÷3

→	→
×240	÷10
÷20	×480
÷100	×2,400
×4,800	÷200
÷30	×720

(Other solutions are possible.)